Review on modeling development for multiscale chemical reactions coupled transport phenomena in solid oxide fuel cells

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Abstract

A literature study is performed to compile the state-of-the-art, as well as future potential, in SOFC modeling. Principles behind various transport processes such as mass, heat, momentum and charge as well as for electrochemical and internal reforming reactions are described. A deeper investigation is made to find out potentials and challenges using a multiscale approach to model solid oxide fuel cells (SOFCs) and combine the accuracy at microscale with the calculation speed at macroscale to design SOFCs, based on a clear understanding of transport phenomena, chemical reactions and functional requirements. Suitable methods are studied to model SOFCs covering various length scales. Coupling methods between different approaches and length scales by multiscale models are outlined. Multiscale modeling increases the understanding for detailed transport phenomena, and can be used to make a correct decision on the specific design and control of operating conditions. It is expected that the development and production costs will be decreased and the energy efficiency be increased (reducing running cost) as the understanding of complex physical phenomena increases. It is concluded that the connection between numerical modeling and experiments is too rare and also that material parameters in most cases are valid only for standard materials and not for the actual SOFC component microstructures.

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### Nomenclature

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<thead>
<tr>
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<tbody>
<tr>
<td>A</td>
<td>active surface area, cm²</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat capacity at constant pressure, J/(kg K)</td>
</tr>
<tr>
<td>D</td>
<td>Darcy number, dimensionless</td>
</tr>
<tr>
<td>$d_p$</td>
<td>electrode particle diameter, m</td>
</tr>
<tr>
<td>$E$</td>
<td>reversible electrochemical cell voltage, V</td>
</tr>
<tr>
<td>$E_a$</td>
<td>activation energy, kJ/mol,</td>
</tr>
<tr>
<td>$F$</td>
<td>Faraday constant, 96485 C/mol</td>
</tr>
<tr>
<td>$F_{ij}$</td>
<td>view factor, dimensionless</td>
</tr>
<tr>
<td>$h_{SG}$</td>
<td>heat transfer coefficient, W/(m² K)</td>
</tr>
<tr>
<td>$h_v$</td>
<td>volume heat transfer coefficient, W/(m³ K)</td>
</tr>
<tr>
<td>$i$</td>
<td>current density, A/cm²</td>
</tr>
<tr>
<td>$i_0$</td>
<td>exchange current density, A/cm²</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity, W/(m K)</td>
</tr>
<tr>
<td>$k_i$</td>
<td>reaction rate constant, mol/(m³ Pa² s)</td>
</tr>
<tr>
<td>$k'$</td>
<td>Boltzmann's constant, J/K</td>
</tr>
<tr>
<td>$k_e$</td>
<td>pre-exponential factor, 1/(Ω m²)</td>
</tr>
<tr>
<td>$K_e$</td>
<td>equilibrium constant, Pa² or dimensionless</td>
</tr>
<tr>
<td>$l_{TPB}$</td>
<td>length of the three-phase boundary, m</td>
</tr>
<tr>
<td>$M_i$</td>
<td>molecular weight of species $i$, kg/mol</td>
</tr>
<tr>
<td>$n_e$</td>
<td>number of electrons transferred per reaction, dimensionless</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number, dimensionless</td>
</tr>
<tr>
<td>p</td>
<td>pressure, Pa, bar</td>
</tr>
<tr>
<td>$q_{ij}$</td>
<td>heat flux, W/m²</td>
</tr>
<tr>
<td>$Q$</td>
<td>source term (heat), W/m³</td>
</tr>
<tr>
<td>$r_i$</td>
<td>chemical reaction rate, mol/(m³ s), mol/(m² s)</td>
</tr>
<tr>
<td>$r_{av}$</td>
<td>average pore radius, m</td>
</tr>
<tr>
<td>$R$</td>
<td>gas constant, 8.314 J/(mol K)</td>
</tr>
<tr>
<td>$R_{el}$</td>
<td>electrolyte, electrons</td>
</tr>
<tr>
<td>$S_i$</td>
<td>source term mass, kg/(m³ s)</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature, K</td>
</tr>
<tr>
<td>$T_P$</td>
<td>viscous stress tensor, N/m²</td>
</tr>
<tr>
<td>$t$</td>
<td>tortuosity, dimensionless</td>
</tr>
<tr>
<td>$u,v$</td>
<td>velocity, m/s</td>
</tr>
<tr>
<td>$w_i$</td>
<td>mass fraction of species $i$, kg/kg</td>
</tr>
<tr>
<td>$x_j$</td>
<td>molar fraction of species $j$, mol/mol</td>
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### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>CFD</td>
<td>computational fluid dynamics</td>
</tr>
<tr>
<td>CM</td>
<td>continuum methods</td>
</tr>
<tr>
<td>DFT</td>
<td>Density Functional Theory</td>
</tr>
<tr>
<td>DCM</td>
<td>Dusty-Gas model</td>
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<tr>
<td>FC</td>
<td>fuel cell</td>
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<tr>
<td>FEM</td>
<td>finite element method</td>
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<tr>
<td>FVM</td>
<td>finite volume method</td>
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<tr>
<td>HCR</td>
<td>heterogeneous reaction mechanism</td>
</tr>
<tr>
<td>HT</td>
<td>high temperature</td>
</tr>
<tr>
<td>IEA</td>
<td>International Energy Agency</td>
</tr>
<tr>
<td>IT</td>
<td>intermediate temperature</td>
</tr>
<tr>
<td>LBM</td>
<td>Lattice-Boltzmann method</td>
</tr>
<tr>
<td>LSM</td>
<td>strontium doped lanthanum manganite</td>
</tr>
<tr>
<td>LTE</td>
<td>local temperature equilibrium</td>
</tr>
<tr>
<td>LTNE</td>
<td>local temperature non-equilibrium</td>
</tr>
<tr>
<td>MC</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>MD</td>
<td>Molecular Dynamics</td>
</tr>
<tr>
<td>PWC</td>
<td>point-wise coupling methodology</td>
</tr>
<tr>
<td>SA</td>
<td>surface area</td>
</tr>
<tr>
<td>SMM</td>
<td>Stefan–Maxwell model</td>
</tr>
<tr>
<td>SOFC</td>
<td>solid oxide fuel cell</td>
</tr>
<tr>
<td>TPF</td>
<td>three-phase boundary</td>
</tr>
<tr>
<td>YSZ</td>
<td>yttria-stabilized zirconia</td>
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### Chemicals

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>CH₄</td>
<td>methane</td>
</tr>
<tr>
<td>CO</td>
<td>carbon monoxide</td>
</tr>
<tr>
<td>CO₂</td>
<td>carbon dioxide</td>
</tr>
<tr>
<td>e</td>
<td>electron</td>
</tr>
<tr>
<td>H₂</td>
<td>hydrogen</td>
</tr>
<tr>
<td>H₂O</td>
<td>water</td>
</tr>
<tr>
<td>N₂</td>
<td>nitrogen</td>
</tr>
<tr>
<td>NH₃</td>
<td>ammonia</td>
</tr>
<tr>
<td>Ni</td>
<td>nickel</td>
</tr>
<tr>
<td>O₂</td>
<td>oxygen</td>
</tr>
<tr>
<td>OH⁻</td>
<td>hydroxyl ion</td>
</tr>
<tr>
<td>V₂O₅</td>
<td>lattice oxygen</td>
</tr>
</tbody>
</table>

### Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>absorptivity, m²/mol</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>emissivity, dimensionless</td>
</tr>
<tr>
<td>$\phi$</td>
<td>porosity, dimensionless</td>
</tr>
<tr>
<td>$\eta$</td>
<td>over-potential, V</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>permeability, m²</td>
</tr>
<tr>
<td>$\kappa_{av}$</td>
<td>deviation from thermodynamic equilibrium, Pa s</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity, Pa s</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density, kg/m³</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>ionic/electronic conductivity, Ω⁻¹ m⁻¹</td>
</tr>
<tr>
<td>$\phi$</td>
<td>charge potential</td>
</tr>
</tbody>
</table>

### Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>a</td>
<td>anode</td>
</tr>
<tr>
<td>act</td>
<td>activation polarization</td>
</tr>
<tr>
<td>c</td>
<td>cathode</td>
</tr>
</tbody>
</table>
1. Introduction

Fuel cells directly convert the free energy of a chemical reactant to electrical energy and heat. This is different from a conventional thermal power plant, where the fuel is oxidized in a combustion process combined with a conversion process (thermal–mechanical–electrical energy), that takes place after the combustion [1]. If pure hydrogen is used, no pollution of air and environment occurs at all, because the output from the fuel cells is electricity, heat and water. Fuel cells do not store energy as batteries do [2]. A fuel cell consists of two electrodes: one anode for fuel and one cathode for oxidant. The electrodes are separated by the electrolyte and connected into an electrically conducting circuit. A gas or liquid, with fuel or oxidant, is transported to the electrode, which should be permeable via a porous structure. Unit cells are further organized into stacks to supply the required electricity [3].

The solid oxide fuel cell (SOFC) electrolyte is non porous ceramic, normally Y2O3 stabilized ZrO2 (YSZ). At an operating temperature between 600 and 1000 °C, the ceramic electrolyte becomes non-conductive for electrons, but conductive to oxygen ions. Cathodes are mostly made from electronically conducting oxides or mixed electronically conducting and ion-conduction ceramics. The anode consists normally of nickel/yttria stabilized zirconia (Ni/YSZ) cermet. SOFCs can be designed with planar, tubular or monolithic structures. The planar design is normally more compact, compared to the tubular design, i.e., a higher volume of specific power is achieved. Tubular and planar SOFCs can be either electrode-, anode- or cathode-supported. A electrode-supported SOFC has thin anode and cathode (~50 µm), and the thickness of the electrolyte is more than 100 µm. This design works preferably at temperatures around 1000 °C. In an electrode-supported SOFC either the anode (anode-supported) or the cathode (cathode-supported) is thick enough to serve as the supporting substrate for cell fabrication, normally between 0.3 and 1.5 mm. The electrolyte is in this configuration very thin, and the operating temperature can be reduced to an intermediate range [4].

The current trend is to decrease the operating temperature. Low temperature (LT)-SOFCs in the range of 300–600 °C is under development, the challenge is to increase the ionic conductivity in the electrolyte. Low temperatures make it possible to use cheaper materials throughout the fuel cell system. An approach with material development on the nanoscale is expected to be very promising [5].

As an alternative to the conventional SOFCs (SOFC-O) with electrolyte of YSZ, the concept of proton ceramic electrolyte (such as BaCeO3 based ceramics) has been developed (SOFC-H). Complete fuel utilization is possible, since the water is produced in the cathode (instead of in the anode) and no dilution of the fuel occurs. Thermodynamic analysis has shown that a higher theoretical efficiency can be reached, compared to conventional SOFC-O [6].

The fuel cell is not a new invention, since the electrochemical process was discovered already in 1838–1839. The interest in fuel cells has been growing exponentially, which is evident from the amount of published scientific papers, after year 2000 [7]. Among various types of fuel cells (FCs), the SOFC has attained significant interest due to its high efficiency and low emissions of pollutants to the environment. High temperature operation offers many advantages, such as high electrochemical reaction rate, flexibility of using various fuels and toleration of impurities [8]. Fuel cell systems are still immature technology in early phases of development, as can be noted due to lack of a dominant design, few commercial systems and a low market demand. The creation of strategic niche markets and search for early market niches are of a vital importance for the further development. It is expected that mass production will start when a dominant design is found, and then production cost will significantly decrease due to the economy of scale [7].

SOFCs can work with a variety of fuels, e.g., hydrogen, carbon monoxide, methane, ammonia and combinations of these [9,10]. Oxygen is reduced in the cathode (Eq. (1)). The oxygen ions are transported through the electrolyte, but the electrons are prevented to pass through the electrolyte. The electrochemical reactions (Eqs. (2) and (3)) take place at the anodic three-phase boundary (TPB). Methane needs to be reformed (Eq. (4)) before the electrochemical reactions [11]. Carbon monoxide can be oxidized in the electrochemical reaction (Eq. (3)) but can also react with water (Eq. (5)). Ammonia is converted to hydrogen (Eq. (6)). The reactions described here are the overall reactions, more detailed reaction mechanisms can be found in [11–12]. Note that methane and ammonia are not participating in the electrochemical reactions at the anodic TPB, methane is catalytically converted, within the anode, into carbon monoxide and hydrogen and ammonia into nitrogen and hydrogen. Hydrogen and carbon monoxide are used as fuel in the electrochemical reactions [10,12].

\[
\begin{align*}
O_2 + 4e^- & \iff 2O^2- \quad (1) \\
H_2 + O^2- & \iff H_2O + 2e^- \quad (2) \\
CO + O^2- & \iff CO_2 + 2e^- \quad (3) \\
CH_4 + H_2O & \iff 3H_2 + CO \quad (4) \\
CO + H_2O & \iff H_2 + CO_2 \quad (5) \\
2NH_3 & \iff N_2 + 3H_2 \quad (6)
\end{align*}
\]

A literature review is conducted to find out what methods have been developed to model SOFCs, arranged according to length scales. The mechanisms behind the transport processes within SOFCs are outlined, in terms of momentum-, mass-, heat- and charge transport. Approaches to model electrochemical as well as internal reforming reactions are discussed in detail. Coupling between different methods, length scales and multiscale model integration are outlined. SOFC microscale models correspond in many cases to the atom or molecular level. The finite element method (FEM) and finite volume method (FVM) are used to model SOFCs at the macroscale level. Multiscale modeling is found to be a promising tool for fuel cell research. COMSOL Multiphysics, based on the FEM, as well as FLUENT, based on the FVM, are examples of commercial codes for analysis of coupling different physical models at different scales. Multiscale modeling increases the understanding for detailed transport phenomena, and can be used to make a correct decision on the specific design and control of operating conditions. Note that this paper reviews SOFC component and the cell models, and an introduction to SOFC stack and system models can be found in [13] and [14], respectively.

2. Transport phenomena mechanisms

SOFCs can be examined from different points of view: as an electrochemical generator in a viewpoint of electrochemical reactions at continuum level, as a heat and mass exchanger in a perspective of fluid dynamics and transport phenomena, or as a chemical reactor in viewpoints of chemical reactions depending on fuel composition and heat effects associated with the electrochemical conversion [4].

The amount of fuel gases transported to the active surface for the electrochemical reactions are governed by different parame-
Mass transport in the electrodes occurs in the gas phase, integrated with the chemical reforming reactions at the solid active surface. The electrodes are porous and mass transfer is dominated by gas diffusion [16]. The interconnect can be assumed to be impermeable for gases. Electron transport effect on the mass transfer needs to be considered since the current is collected [4].

Fick’s model is the simplest diffusion model used for dilute or binary systems [26]. In the literature the Stefan–Maxwell model is commonly used to calculate the diffusion in a multi-component system. In some references the Stefan–Maxwell model is combined with the Knudsen diffusion term (frequently called the Dusty-Gas model or extended Stefan–Maxwell equation) [18,20,27–30] to predict the collision effects between the gas molecules and the solid active surface. In other models this effect is neglected [16].

Dusty-Gas model (DGM), Ficks model (FM) and Stefan–Maxwell model (SMM) are developed in [19], in order to predict the concentration over-potential inside an SOFC anode. DGM and FM consider molecular diffusion, Knudsen diffusion and the effect of a finite pressure gradient. The flux ratio in DGM depends on the square-root of the gas molecular weight, but it does not for FM. Explicit analytical expressions describing fluxes can be used in FM. The SMM can be seen as a simpler model since it does not consider the Knudsen diffusion. DGM is the most appropriate model for H2–H2O and CO–CO2 system. However, it is only used when the operating current density is high. Ni et al. [12,31] studied the effect of microstructural grading on SOFC performance with a DGM to analyze the coupled phenomena of mass transfer and electrochemical reactions in the SOFC electrodes.

To account for the increased diffusion length due to the tortuous paths of real pores in the porous materials, different approaches can be found in the literature [25]:

\[
D_i^{\text{eff}} = \frac{q_f^{i}}{\rho} \cdot \frac{D_i}{r} \\
D_i^{\text{eff}} = \frac{\rho}{T} \cdot D_i
\]

where \(D_i\) is the Maxwell–Stefan binary diffusion coefficient, \(D_i^{\text{eff}}\) the effective Maxwell–Stefan binary diffusion coefficient in the porous medium, \(\rho\) the porosity and \(r\) the tortuosity. Similar expressions can be found for molecular diffusion coefficients in porous materials [29,30]. Tortuosities up to the range of 10–17 can be found in the literature. These values are too high considering that the range for porous sintered ceramics is reported to be 2–10 [29].

The Maxwell–Stefan equation can be formulated as [32]:

\[
\nabla \left( -\rho \cdot w_j \sum D_{ij}^{\text{eff}} \cdot \nabla x_j + \left( x_j - w_j \right) \frac{\nabla p}{\rho} u - D_{ij}^{\text{eff}} \cdot \frac{\nabla T}{T} \right) \\
+ \rho \cdot u \cdot \nabla w_j = S_i
\]

where \(w\) is the mass fraction, \(x\) the mole fraction, \(D_{ij}^{\text{eff}}\) the thermal diffusion coefficient and \(S_i\) the source term due to chemical reactions. The effective diffusion coefficient \(D_{ij}^{\text{eff}}\) equals the molecular diffusion coefficient \(D_{ij}\) in the gas channels.

Mass or molar fractions need, for the inlet fuel- and air flows, to be defined at the cell level. The outlet conditions are usually defined as convective fluxes. The effects on mass transport from the electrochemical reactions are either defined as source terms in the governing equations or as interface conditions, as discussed previously [33].

For instance, Andersson [33] has developed a model considering fluid flow, mass- and heat transfer for a single cell IT-SOFC. The mole fraction of oxygen (Fig. 1) decreases along the flow direction in the air channel and the cathode. There is a concentration difference in the y-direction as well, that forces the flow towards the cathode/electrolyte interface. However, it is hard to recognize in the air channel as the cell length is 400 times bigger than the air channel height.

Fig. 1. Mole fraction of oxygen in the air channel and cathode for a single cell IT-SOFC [33].
racy. The Knudsen diffusion is used when the pores are small in
comparison to the mean free path of the gas. In this case, molecules
 collide more often with the pore walls than with other molecules.
The Knudsen diffusion coefficient can be calculated using a kinetic
theory that relates the diameter of the pore and the mean free path
of the gas according to [30]:

\[
D_k = \frac{u \cdot r^2}{6}
\]

(12)

where \(D_k\) is the Knudsen diffusion coefficient, \(r\) is the average pore
radius and \(u\) is the velocity of the gas molecules. If the pores are
straight and circular, the diffusion coefficient of the component \(i\)
is [30]:

\[
D_{ik} = 97.0 \cdot r^2 \cdot \sqrt{\frac{T}{M_i}}
\]

(13)

where \(D_{ik}\) is the Knudsen diffusion coefficient for molecule \(i\), \(T\) the
temperature and \(M_i\) the molecular weight. To calculate the average
 pore radius (\(r^2\)), the surface area of the porous solid and the porosity
are used [30]:

\[
r^2 = \frac{2 \cdot \varphi}{SA \cdot \rho_b}
\]

(14)

where \(SA\) is the surface area of the porous solid and \(\rho_b\) the bulk den-
sity of the solid particle. It is possible to account for the tortuous
path of the molecule, by calculating the effective Knudsen diffusion
coefficient [30]:

\[
D_{ik, eff} = D_{ik} \cdot \left( \frac{\varphi_i}{\varphi} \right)
\]

(15)

where \(D_{ik, eff}\) is the effective Knudsen diffusion coefficient. Molecular
diffusion and Knudsen diffusion may appear at the same time and
can be calculated as [26,30]:

\[
\frac{1}{D_{i, eff}} = \frac{1}{D_{i, eff}} + \frac{1}{D_{ik, eff}}
\]

(16)

2.2. Momentum transport

The fuel cell manifolds should be designed with the aim to
achieve uniform fluid distribution between different unit cells
and a stable stack operation. The flow field in a unit cell may be di-
vided into three parts, inlet manifold, cell gas channels (fuel and
air) and outlet manifold. The inlet manifold for a single cell is a part
of the stack inlet manifold, the air (or fuel) is directed into the sin-
gle cell and the excess is transported to the next cells. For a stack
with a given number of single cells, the geometric parameters that
are influential to the stack flow uniformity are: the height of the
repeating unit cell, the height and length of the gas channels, the
inlet manifold width and the outlet manifold width [34].

The effects of the geometric parameters on the stack flow uni-
formity are normally understandable in terms of their effects on
the relative dominances between the pressure changes in the gas
channels and manifolds. According to a study in [34], the fuel flow
distribution is more uniform than the air flow distribution for a gi-
ven stack, due to the fact that the fuel flow velocity is small or in
the laminar region, with a small pressure change in the manifold.
The results also show that the flow distribution for stacks with a
large number of cells can be far from ideal conditions. It is revealed
that the ratio between inlet and outlet manifold widths is a key
parameter for the flow distribution, and the optimal ratio increases
with the number of cells in the stack.

The gases flow inside the fuel cell components, such as in the air
and fuel channels and to the porous electrodes. The physics of lim-
inar and turbulent incompressible flow are well described by the
Navier–Stokes equations. It is common practice to assume laminar
flow for fuel cell gas channels due to the low velocities, which de-
creases the computational cost significantly [25]. The Darcy equa-
tion describes the balance in the porous electrodes between the
force from the pressure gradient and the frictional resistance from
the solid material. It should be noted that the Darcy equation ex-
presses the flow in the porous structure well away from the walls.
A traditional modeling approach for such a system consists of solv-
ing the Darcy’s equation in the porous medium and the Navier–
Stokes equations in the channels separately. The problem with
such an approach is to define interfacial conditions at the interface
between the two domains. It is hard to define this tangential velocity
component. To avoid this problem the Darcy–Brinkman equa-
tion is introduced and solved for the gas flow in the fuel and air
channels and in the porous media (electrodes) [20,32,35].

The Darcy–Brinkman equation (Eq. (17)) is transformed into the
standard Navier–Stokes equation when \((k \to \infty)\) and \((\varphi = 1)\), and
into the Darcy equation as \((Da \to 0)\). \(Da\) is the Darcy number. The
derivation of the Navier–Stokes equation and Darcy equation from
Darcy–Brinkman equation can be found in [35].

\[
\left(\frac{\eta}{k} \cdot \rho \cdot \nabla u \right) \cdot u - \left( -p + \frac{1}{\varphi} \left( T - (\lambda - \kappa_{dy}) |\nabla u| \right) \right) = F
\]

(17)

where \(F\) is the volume force vector, \(k\) the permeability of the porous
medium, \(\varphi\) the porosity, \(\mu\) the dynamic viscosity, \(u\) the velocity vec-
tor and \(T\) the viscous stress tensor \((T = \eta(\nabla u + (\nabla u)^\top))\). \(\kappa_{dy}\) is the deviation from thermodynamic equilibrium. When this term is zero,
the fluid particles are in equilibrium with their surroundings. \(\lambda\) is the second viscosity and is, for gases, normally assumed as: \(\lambda = -2\beta_{dy}\) [36].

When a single cell is modeled, a velocity (profile) is defined at the
air- and fuel channel inlets (frequently as a laminar flow pro-
file) and pressure at the air- and fuel channel outlets. The effects
from the electrochemical reactions are either defined as source
terms in the governing equations or as interface conditions as dis-
cussed previously [33,37].

2.3. Heat transport

The heat transfer inside SOFCs includes various aspects such as
convective heat transfer between the solid surfaces and the gas
streams, conductive heat transfer in solid and porous structures.
Heat generation occurs due to the electrochemical reactions at
TPB between the electrolyte and electrodes [38], the current flow
(ohmic polarization) and the internal reforming reactions of carbon
monoxide at the porous anode and in the fuel channel [39]. The
steam reforming reaction of methane in the porous anode is endo-
thermic, and heat is then consumed [39]. Accurate temperature
prediction distribution within SOFCs is essential for predicting and
optimizing the overall cell performance as well as avoiding thermo-
mechanical degradation [40]. Most of the heat within SOF-
Cs is generated near the electrode/electrolyte interface and is dis-
sipated by: (1) conduction in the solid matrix, (2) heat transfer
from the solid to the gas phase by convection within the pores,
and (3) advection of the gas through the micro-pores to the flow
channel [40].

Effective transport parameters for the porous material need to be
calculated when a local temperature equilibrium (LTE) ap-
proach (assumes the same temperature for gas- and solid phase)
is used. The thermal conductivity \((k_{eff})\) and specific heat \((c_{p,eff})\)
can be specified as [18]:

\[
k_{eff} = \varphi \cdot k_f + (1 - \varphi) \cdot k_s
\]

(18)

\[
c_{p,eff} = \varphi \cdot c_{pf} + (1 - \varphi) \cdot c_{ps}
\]

(19)
where $\phi$ is the porosity, $\text{eff}$ means effective, $s$ solid and $f$ fluid (gas) phase. The temperature distribution can be predicted by [32]:

$$\nabla ( - k_{\text{eff}} \cdot \nabla T ) = Q - \rho_{\text{eff}} \cdot c_{p,\text{eff}} \cdot \frac{1}{\partial T / \partial x}$$

(20)

where $T$ is the temperature (same temperature is assumed for solid- and gas phase), and $Q$ the source term (due to the chemical- and electrochemical reactions).

2.3.1. Local temperature non-equilibrium (LTNE) approach

A very common method in SOFC modeling is to employ LTE [18,26,40]. However, some typical conditions found in the porous SOFC electrodes bring this assumption into question: (1) very low Reynolds number flow, (2) presence of volumetric heat generation, and (3) large difference in thermal conductivities between the gas- and solid phases. A local temperature non-equilibrium (LTNE) approach is developed in [40] to predict the temperature difference between the solid- and gas phases within the porous electrodes. Heat is (according to the LTNE approach) transferred between the phases at the solid material surfaces in the porous electrodes. The general heat conduction equation is used to calculate the temperature distribution for the solid matrix in the porous electrodes [32]:

$$\nabla ( - k_s \cdot \nabla T_s ) = Q_s$$

(21)

where $k_s$ is the thermal conductivity of the solid, $T_s$ the temperature in the solid phase and $Q_s$ the heat source (heat transfer between the solid- and gas phases, the heat generation due to the ohmic polarization and due to the internal reforming reactions). The heat generation due to electrochemical reactions, concentration polarization and activation polarization can be defined either as a source term or as an interface condition. Note that part of the heat generation/consumption enters into the solid phase and the remaining part to the gas phase. Knowledge about this ratio is still pore. The temperature difference for the solid matrix in the porous electrodes can be defined by symmetries, when the cell is assumed to be surrounded by the top and bottom of the cells are (for a cell model) normally defined by symmetries, when the cell is assumed to be surrounded by other similar cells with the same temperature distribution [33].

Most cell level models define the temperature at the air and fuel channel inlets and convective flux at the outlets. The boundaries at the top and bottom of the cells are (for a cell model) normally defined by symmetries, when the cell is assumed to be surrounded by other similar cells with the same temperature distribution [33].

For example, Andersson [33] uses a LTNE approach to calculate the temperature distribution for a single cell IT-SOFC. The temperature increases along the $x$-direction (the main flow direction), as seen in Fig. 2. The temperature difference in the $y$-direction inside the air channel occurs because the convective heat flux is bigger in the air channel (compared to the fuel channel) due to the relatively larger gas flow rate. It is revealed that the temperature difference between the phases is less than 1 K for the studied case (not shown here).

2.3.2. Thermal radiation

SOFCs are operating at a high temperature and thus thermal radiation should be involved for an accurate heat balance. Heat exchange between the surfaces of the air and fuel channels, radiation between the surfaces of the channels and flowing gases (especially water and carbon dioxide) and radiation heat loss from the stack to the environment should be taken into account for effective thermal management of the fuel cell stack. The thermal radiation between the surfaces may be calculated as [42]:

$$q_i = \sum_{j=1}^{n} A_i k_i(T_i^4 - T_j^4)$$

(25)

where the radiation leaves surface $i$ as a result of its interaction with surface $j$, $T$ is the temperature (in Kelvin), $k$ is the Boltzmann constant and $F$ is the view factor. The net rate between the channel surfaces (assumed to have blackbody conditions) and the channel gases can be expressed as [42]:

$$q_{net} = A_i k_i(T_i^4 - \varepsilon_i T_i^4)$$

(26)

where $A$ is the active surface area, $\varepsilon$ the emissivity and $\alpha$ the absorptivity.

2.4. Charge transport

The SOFC electrolyte has two functions: to transport oxide ions from the electrolyte to the anode and to block electron flow from the anode to the cathode [11]. The flow of electronic charges through external circuit balances the flow of the ionic charge through the electrolyte and electrical power is produced [43]. This transport can be described by considering the ion transport from the conservation of charge as [26]:

$$\nabla \cdot i = 0 = \nabla \cdot i_{io} + \nabla \cdot i_{el}$$

(27)

$$-i_{io} = \nabla \cdot i_{el}$$

(28)

$$i_{el} = -\sigma_{\text{eff}} \nabla \phi_{el}$$

(29)

Eq. (29) is Ohm’s law, $i_{io}$ and $i_{el}$ are charge fluxes for ions and electrons, respectively, and $\phi_{el}$ is the potential in the electrolyte. The interfaces/boundaries can be defined as electric potential ($V = V_{io}$) or ground conditions ($V = 0$). The effects of the electrochemical reactions are either defined as source terms in the governing equations or as interface conditions.
at the electrolyte/electrode interface, as previously discussed [32]. Bessler et al. [37] assume that the electrode component has a constant electric potential.

The Nernst potential is usually calculated as the sum of the potential differences across the anode and cathode [26]:

$$ E = \Delta \phi_a + \Delta \phi_c $$

(30)

where $E$ is the reversible electrochemical cell voltage and $\phi$ the charge potential.

### 2.5. Interaction issues

The mass-, heat-, momentum- and charge transport and the chemical reactions are dependent on each other. The fluid properties and the flow field (momentum transport) depend on the temperature and the species concentrations. The (electro-) chemical reaction rates depend on temperature, species concentrations and available surface areas for catalytic reactions. The chemical reactions generate and consume heat, i.e., the temperature distribution depends on the chemical reaction rates, as well as on the solid and the gas properties (for example the heat capacity and the conductivity). All these dependences require that the governing equations are solved coupled [1].

### 3. Microcatalytic reaction mechanisms

#### 3.1. Electrochemical reactions

Electrochemical reactions occur at the TPB, i.e., the region where the electrode, electrolyte and gas phase meet. Ions migrate in the ionic phase, conduction of electrons occurs in the electronic phase and transport of gas molecules takes place in the porous part of the electrodes. A larger TPB area gives more reaction sites (lower activation polarization in the electrodes) [16]. Note that the TPB needs to be connected to the rest of the structure, i.e., the pores need to be connected through the surrounding pore network to the fuel/air stream, the nickel phase must be connected to the current collector and the YSZ phase to the bulk YSZ electrolyte [29].

The total pressure is lower close to TPB, compared to other parts of the cathode due to the consumption of oxygen molecules at TPBs. This gradient makes the transport of oxygen from the channel towards the electrolyte easier. The TPB area in the electrode depends on the particle diameter. A reduction of the particle diameter increases the TPB area, at the same time the Knudsen diffusivity and the flow permeability are reduced. In [16] it is found that most of the electrochemical reaction occurs within 10 μm (from the electrolyte interface) for the anode and 50 μm for the cathode when the mean particle diameter is 1 μm [16].

At the interface between the electrode and electrolyte the Butler–Volmer equation can be used to calculate the volumetric current density [30]:

$$ i = i_0 \left\{ \exp \left( \frac{\beta \cdot \eta_{\text{int}}}{R \cdot T} \right) - \exp \left( -\left(1 - \beta \right) \cdot \frac{\eta_{\text{int}}}{R \cdot T} \right) \right\} $$

(31)

where $F$ is the Faraday constant, $R$ the ideal gas constant and $\beta$ the transfer coefficient, usually assumed to be 0.5. The Butler–Volmer equation can often be expressed as [30]:

$$ i = 2 \cdot i_0 \cdot \sinh \left( \frac{\eta_{\text{act}}}{2 \cdot R \cdot T} \right) $$

(32)

$$ \eta_{\text{act}} = \frac{2 \cdot R \cdot T}{n_e \cdot F} \sinh^{-1} \left( \frac{i}{2 \cdot i_0} \right) $$

(33)

$$ i_0 = \frac{R \cdot T}{n_e \cdot F} \cdot k^0 \cdot \exp \left( -\frac{E_a}{R \cdot T} \right) $$

(34)

where $i_0$ is the exchange current density, $k^0$ the pre-exponential factor and $E$ the activation energy.

The charge transfer processes are ones of the least understood aspects of fuel cell chemistry [29,44]. It is expected that understanding the mechanism of the surface reactions that occur close to the anode TPB is important for the future advances of the SOFC development [44]. Most models in the literature assume a single global charge transfer reaction, leading to the Butler–Volmer equation (Eq. (31)). Adsorption/desorption, surface diffusion, the formation of hydroxyl and a charge transfer reaction are all found to be feasible rate-limiting steps in an SOFC (anode) model [29].

A five elementary reaction mechanism for the electrochemical reactions used by Lee et al. [29], among others, reads:

$$ \text{H}_2(g) + 2(\text{Ni}) \Leftrightarrow 2\text{H}(\text{Ni}) $$

(35)

$$ \text{H}(\text{Ni}) + \text{O}_2(\text{YSZ}) \Leftrightarrow (\text{Ni}) + \text{OH}^-(\text{YSZ}) + e^- (\text{Ni}) $$

(36)

$$ \text{H}(\text{Ni}) + \text{OH}^- (\text{YSZ}) \Leftrightarrow (\text{Ni}) + \text{H}_2\text{O}(\text{YSZ}) + e^- (\text{Ni}) $$

(37)

$$ \text{H}_2\text{O}(\text{YSZ}) \Leftrightarrow \text{H}_2\text{O}(g) + (\text{YSZ}) $$

(38)

$$ \text{O}_2^\text{a}(\text{YSZ}) + (\text{YSZ}) \Leftrightarrow \text{O}_2^\text{a}(\text{YSZ}) + \text{V}_0(\text{YSZ}) $$

(39)

Hydrogen is assumed to be adsorbed only at the nickel (Ni) surface and other surface species at the electrolyte surface (YSZ). Interactions between (1) adsorbed atomic hydrogen H(\text{Ni}), an empty surface site (\text{Ni}) and an electron e^- (\text{Ni}), (2) a lattice oxygen O_2^\text{a}(\text{YSZ}) and an oxygen vacancy, V_0(\text{YSZ}) within the YSZ electrolyte, and (3) hydroxyl ion OH^- (YSZ), water H_2\text{O}(YSZ), oxygen ion O_2^\text{a}(YSZ) and empty YSZ sites are considered in this reaction mechanism (Eqs. (35)–(39)). The expression for the exchange current density depends on the assumption of which equation is the rate-limiting one, as shown in Table 1. Note that different expressions can be obtained, based on the dependence on the partial pressure of the product and reactants, anodic or cathodic reaction coefficient (k_i or k_p) and the equilibrium constants (K) for the respective reactions. Determination of the actual rate-limiting reactions among Eqs. (35)–(38) requires a very careful analysis and fitting of experimental data to the different theoretical derivations. Also the anodic and cathodic charge-transfer coefficients in the Butler–Volmer equation depends on the assumption of the rate-limiting equations. The hydrogen adsorption rate can be described as: $i_i = i_0 \cdot p_i$, where $i_0$ can be seen as an adjustable parameter (connected to the exchange current density) to match the experimentally observed performance [29].

Vogler et al. [44] studied the electrochemical hydrogen oxidation reaction at nickel/YSZ anodes and considered Eqs. (35)–(39) but also the YSZ surface reaction between water and oxygen ions, Ni surface reaction for water and hydroxyl ions, and five additional charge transfer reactions. Their computational model is based on elementary physics and chemical processes (without the assumption of a specific rate-determining step). Models and experiments

### Table 1

<table>
<thead>
<tr>
<th>Rate limiting reaction</th>
<th>Exchange current density ($i_0$)</th>
<th>Hydrogen adsorption ($i_0$) $p_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. (35) $i_0 \cdot p_{i_0}$</td>
<td>$2F_i \gamma_T \gamma_{E,SOFC} \gamma_{E,SOFC} \gamma_{E,SOFC}$ $p_{i_0}$</td>
<td></td>
</tr>
<tr>
<td>Eq. (36) $i_0 \cdot \gamma_T \gamma_{E,SOFC} \gamma_{E,SOFC} \gamma_{E,SOFC}$ $p_{i_0}$</td>
<td>$2F_i \gamma_T \gamma_{E,SOFC} \gamma_{E,SOFC}$ $p_{i_0}$</td>
<td></td>
</tr>
<tr>
<td>Eq. (37) $i_0 \cdot \gamma_T \gamma_{E,SOFC} \gamma_{E,SOFC}$ $p_{i_0}$</td>
<td>$2F_i \gamma_T \gamma_{E,SOFC} \gamma_{E,SOFC}$ $p_{i_0}$</td>
<td></td>
</tr>
<tr>
<td>Eq. (38) $i_0 \cdot \gamma_T \gamma_{E,SOFC} \gamma_{E,SOFC}$ $p_{i_0}$</td>
<td>$2F_i \gamma_T \gamma_{E,SOFC} \gamma_{E,SOFC}$ $p_{i_0}$</td>
<td></td>
</tr>
</tbody>
</table>
are compared to determine the anode performance and to estimate the pre-exponential factors and the activation energies. Only the forward-rate coefficients from the experimental and literature data are used and then the reverse-rate constants are calculated according to thermodynamic consistency. Seven possible different combinations of one or two elementary charge transfer reactions are investigated and compared to the experimental data. It is found that the hydrogen spillover mechanisms (Eq. (37) together with Eq. (40)) have best agreement with the experimental data. The calculated activation energy (using experimental data) is higher than expected. This may be explained by an unfavorable geometry at atomic scale.

\[
H(Ni) + O^2^- (YSZ) \leftrightarrow (Ni) + OH^- (YSZ) + e^- (Ni)
\]  

A high TPB length (or area) is required for high electrochemical performance, and of crucial importance for the fuel cell performance. Janardhanan et al. [45] concluded that the maximum volume specific TPB length is achieved at 50% porosity (while other parameters are fixed). A mathematical model is developed in [45] based on porosity, particle diameter and volume fraction of the ionic- and electronic phase. The calculation can be based on both uniform and non-uniform particle size distributions.

Asinari et al. [46] uses the Lattice-Boltzmann method to construct a 3D microscopic topology, where the TPB area is calculated. Both the electron and the ion dynamics are included in the numerical model. In order to realize an electrochemical reaction the electron conducting cell (EEC) must be in contact with the electron sink, i.e., at least one connection path must exist between the EEC in question and the collection grid.

Martinez and Brouwer [47] have developed a Monte Carlo percolation model to be able to characterize the factors controlling the TPB formation within an SOFC electrode. The model considers ionic and electronic conductor, gas phase percolation, competition between percolation of gas and electronically conducting phases. It is found that physical processes such as sorbate transport significantly affect the TPB formation. It is also concluded that the TPB formation is most effective when the electronic volume fraction is quite low.

The contribution to the gas species distributions (in mol/(m²s)) by the electrochemical reactions is given as [25,27,48]:

\[
r_{H2} = \frac{-i}{2 F} \\
r_{H2O} = \frac{i}{2 F} \\
r_{O2} = \frac{-i}{4 F}
\]

where \(i\) is the current density and \(F\) the Faraday constant.

### 3.2. Internal reforming reactions

Internal reforming reactions inside SOFC porous anodes enable the conversion of hydrocarbon fuels (and water) into hydrogen and carbon monoxide, and further carbon monoxide (and water) to hydrogen. The heat, needed by the steam reforming reaction, is generated in the electrochemical reactions at the active surface (TPB) between the porous anode/electrolyte and cathode/electrolyte. A good heat transfer (in terms of short heat transfer distance between heat generation and consumption within the cell) can be achieved and the conversion efficiency is increased. Hydrogen and carbon monoxide can be oxidized as soon as they are produced by the reforming reactions, and steam produced by the electrochemical reaction (Eq. (2)) can be used in the reforming reactions (Eqs. (4) and (5)) [15,49].

The steam reforming of hydrocarbon fuels (Eqs. (4) and (5)) could either take place before the fuel cell stack (in an external pre-reformer) or inside the cell in the anode (internal reforming). A pre-reformer needs extra added steam, since it cannot use the steam generated in the electrochemical reactions. The internal steam reforming reaction takes place over ceramic-supported nickel catalysts and decreases the requirement for cell cooling (less surplus of air). Less steam is needed for the reforming reactions and finally it offers advantages with respect to the capital cost. Up to half of the heat produced by the oxidation reaction (exo- thermic) could be “consumed” by the steam reforming process. This would improve the system electrical efficiency [39].

The probability for carbon depositions depends on the steam/methane ratio. It has been well established that the key reactions occur over a surface layer of nickel atoms. If a layer of carbon is allowed to build up and attach to a nickel crystallite rapid catalyst breakdown can occur, due to the graphite formation. It should be noted that hydrocarbons with a longer coal chain than methane have a higher propensity for carbon deposition. To avoid the carbon deposition inside the SOFC anode pre-reforming have to be carried out before the fuel enters into the cell at a lower temperature at which carbon deposition does not occur [50].

In a conventional (high temperature (HT)) SOFC (with nickel content of about 50 vol.% and operating temperature about 1000 °C) the endothermic reaction is very fast. This can result in a temperature drop at the inlet of the stack. The temperature gradient results in thermal tensions, which in the worst case causes mechanical failure of the cells [39]. The problem of the tensions and big temperature gradients close to the inlet could be solved with different approaches:

1. Lowering the operating temperature to an intermediate range to reduce the steam reforming reaction rates [51].
2. Recycling a part of the anode gas to obtain a dilution of the fuel. The rate of reforming reactions decreases, due to decrease in fuel concentration. A 50% recycling results in sufficient steam for the reforming reactions and the cost for a separate water supply is saved [39].
3. The anode material can be designed with the aim of a decreased steam reforming activity. Until now these new SOFC materials (such as iron or copper) have too low electronic conductivity to meet the real world requirements. When nickel is replaced during the fabrication process with for example iron, a less catalytic active anode regarding the reforming activity is constructed. This is in the short term a promising method, and this approach is based on well-established production processes. Other researches replace nickel with copper and the same effect has been reached [39].

The reforming reactions occur in the porous anode. The reaction rates can be described with simplified global expressions (Section 3.2.1) or detailed expressions for surface chemical reactions (Section 3.2.2). Some models in literature are compared in terms of internal reforming reaction mechanisms and involved gas species, as in Table 2. It is found that comparison with experimental data is rare. Most of the models [6,23,52–55] use an equilibrium approach both for the water–gas shift reaction and steam reforming reaction. For the steam reforming reaction a comparison of the kinetic expression with different dependences of water and methane is presented in this table. It is common to use an inlet concentration considering 30% pre-reformed natural gas, which is also specified as International Energy Agency (IEA) conditions [6,23,54–55]. The ratio of water to methane is varied, in [53], to study the effect of gradual vs. direct internal reforming reactions.
3.2.1. Global reforming reaction kinetics

Methane can be converted to hydrogen and carbon monoxide inside the porous anode by the catalytic steam reforming. Carbon monoxide reacts further (inside the anode as well in the fuel channel) with steam to produce hydrogen and carbon dioxide according to the water–gas shift reaction [24,56]. The overall reaction can be written as Eqs. (4) and (5). Hydrogen reacts with oxygen ions at the anodic TPB and generates steam. As hydrogen is consumed and steam is generated, the water–gas shift reaction proceeds towards the right, i.e., more hydrogen is produced. The catalytic reforming reactions occurs at the surface of the porous structure inside the anode [55].

Several kinetic expressions considering the steam reforming reaction are developed in the literature. The reaction order varies significantly between the models, in terms of orders of methane and water (m and n in Eq. (44)), see Table 2 for typical values. The reaction order of methane (m) varies between 0.85 and 1.4. The highest difference in reaction orders is found for water (n), both negative and positive values exist, however, it has been shown that all these findings could be correct for the chosen operating conditions of the experiments. Small steam-to-carbon (SC) ratio gives positive reaction order of water. SC in the order of two yields the reaction orders of water close to zero, and high SC gives negative values [54]. Experimental work has shown that it is possible to change the steam reforming reaction orders by modifying the anode, for example m increased from 0.85 to 1.4 and n decreased from −0.35 to −0.8 when basic compounds are added in Ni–YSZ anode [51]. As seen in Table 2, comparison with experimental data is rare, but it is worth while to note that the reaction orders (m and n) usually originate from fitting experimental data to a kinetic expression.

\[
r_r = k \cdot p_{CH_4}^m \cdot p_{H_2O}^n \cdot \exp \left( -\frac{E_a}{RT} \right)
\]  

(44)

where \( r_r \) is the reaction rate of steam reforming reaction (in mol/s/m²), \( p \) the partial pressure (in bar), \( R \) the ideal gas constant and \( T \) the temperature. The reaction rate is converted to mol/s/m² after multiplying the reaction rate (mol/s/m) with the surface area ratio (SA). The kinetic expression (Eq. (44)) can be compared to the equilibrium approach (Eq. (45)) [54].

\[
r_{r,eq} = k \cdot p_{CH_4} \cdot p_{H_2O} \cdot \left( 1 - \frac{p_{CO} \cdot p_{H_2}}{K_{STR} \cdot p_{CH_4} \cdot p_{H_2O}} \right)
\]  

(45)

where \( K_e \) is the equilibrium constant. The reaction orders in Eq. (45) are the same as the molar proportions for the respective gas species in the global chemical reaction (1 for both methane and water for the forward reaction, and 3 for hydrogen and 1 for carbon monoxide for the backward reaction).

Different approaches for defining the water–gas shift reaction (Eq. (5)) can be found in literature: (1) Global reaction mechanism that considers reaction in the anode only [6,18]; (2) Global reaction mechanism that considers reaction in the anode and in the fuel channel [53,57]. (3) A more advanced reaction mechanism that includes catalytic surface reaction kinetics for steam reforming, water–gas shift reaction and the Boudouard mechanism can be found in [27,58]. It is frequently stated in literature [52–53,57] that the water–gas shift reaction should be considered to be in (or very close to) equilibrium state. When the equilibrium is assumed the reaction velocity can be expressed with an equilibrium-limited shift reaction rate expression, first order in carbon monoxide as [55]:

\[
r_s = k_s \cdot p_{CO} \cdot \left( 1 - \frac{p_{CO} \cdot p_{H_2}}{K_{STR} \cdot p_{CH_4} \cdot p_{H_2O}} \right)
\]  

(46)

where \( r_s \) is the reaction velocity of the water–gas shift reaction, \( k_s \) the pre-exponential factor, \( p \) partial pressure for the respective species and \( K_{STR} \) the temperature dependent equilibrium constant [55].

The concentration of the different gases in the anode (the steam reforming and the water–gas shift reactions) and fuel channel (the water–gas shift reaction) is changing as the above described reactions proceed:

\[
\begin{align*}
 r_{CH_4} &= -r_r \\
 r_{H_2} &= 3 \cdot r_r + r_s \\
 r_{CO} &= r_r - r_s \\
 r_{CO_2} &= r_s \\
 r_{H_2O} &= -r_r - r_s
\end{align*}
\]  

(47–51)

where \( r_i \) is the reaction rate of species \( i \). The catalytic steam reforming rate is expressed by \( r_r \) and the water–gas shift reaction rate by \( r_s \).

Andersson [33] uses a global approach to calculate internal reforming reactions for a single cell SOFC. The gas-phase tempera-

---

Table 2
Comparison of models considering internal reforming reactions.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CH₄</td>
<td>0.09</td>
<td></td>
<td>0.970</td>
<td>0.171</td>
<td>0.171</td>
<td>0.171</td>
<td>0.280</td>
<td>0.171</td>
</tr>
<tr>
<td>CO</td>
<td>0.07</td>
<td>n/a</td>
<td>0</td>
<td>0.029</td>
<td>0.029</td>
<td>0.029</td>
<td>0.005</td>
<td>0.029</td>
</tr>
<tr>
<td>CO₂</td>
<td>0.22</td>
<td>n/a</td>
<td>0</td>
<td>0.044</td>
<td>0.044</td>
<td>0.044</td>
<td>0.030</td>
<td>0.044</td>
</tr>
<tr>
<td>H₂</td>
<td>0.28</td>
<td>n/a</td>
<td>0</td>
<td>0.263</td>
<td>0.263</td>
<td>0.263</td>
<td>0.120</td>
<td>0.263</td>
</tr>
<tr>
<td>H₂O</td>
<td>0.34</td>
<td>n/a</td>
<td>0.030</td>
<td>0.493</td>
<td>0.493</td>
<td>0.493</td>
<td>0.565</td>
<td>0.493</td>
</tr>
</tbody>
</table>

* a The ratio of water to methane (\( x_{H_2O}/x_{CH_4} \)) is varied between 0.01 and 1.

b A comparison study between different equations for reaction rates, in the reference, is made when more than one value is presented [51–52,54].
ture within the cell is presented in Fig. 3. The steam reforming reaction reduces the temperature close to the fuel channel inlet. The temperature is slightly lower on the air side (compared to the fuel side), due to a higher air flow rate. The initial temperature decrease is found to be 15 K in the anode, compared to 14 K in the cathode, 13 K in the fuel channel and 11 K in the air channel. For the case with higher operating temperature a higher reforming reaction rate is expected and also the temperature close to the inlet decreases faster. The mole fraction distribution of methane can be seen in Fig. 4. The concentration difference in the y-direction is due to the consumption of methane in the porous structure. Methane diffuses from the channel towards the porous anode. Note that the fuel channel length is 100 times bigger than the channel height.

### 3.2.2. Elementary surface reaction kinetics

Elementary-kinetic electrochemical modeling of SOFC electrodes is strongly connected to the heterogeneous catalysis and the surface chemistry (as shown in Figs. 5 and 6). Note that information considering the surface coverages are not available when the global approach is applied. Both for the anode and cathode there is (within the literature) a disagreement considering the involved reaction pathways, rate-limiting steps and intermediate species [37,59]. Elementary-kinetic models are normally limited to only the electroactive region, and couplings to transport models are still rare. The electroactive species occurring in the charge transfer reactions are surface-adsorbed intermediates. The surface coverage can be calculated with for example a mean-field approach, where it is assumed that the surface state can be described with average quantities, such as surface coverage, thermodynamic and kinetic adsorbate properties. The composition (segregation phenomena and impurities) and the detailed surface structure (steps, edges and terraces), occurring on the atomic scale, are then not further resolved, when using the mean-field approach. The surface concentration of the different species is normalized to the total concentration of available surface sites, yielding the dimensionless coverage. The surface species are coupled to the gas-phase species via desorption and adsorption reactions, and to bulk-phase species via surface/bulk exchange reactions. The surface species are also participating in the reactions among themselves [37].

An SOFC anode is normally fabricated as a porous metal–ceramic composite, where the gas, ceramic and metal phases occupy roughly 30% of the volume each, the characteristic pore dimensions are in the order of 1 μm [61]. Knowledge of the catalytic reaction mechanism considering reforming of hydrocarbons is counted as a key importance for designing an anode material with a high efficiency and a long life length [62].

Janardhanan and Deutschmann [63] have developed a multi-step heterogeneous reaction mechanism for Ni catalysts. The mechanism consists of 42 reactions, 6 gas-phase species and 12 surface adsorbed species. The mechanism is elementary in nature and covers the global aspects of the reforming, the water–gas shift and the Boudouard reactions. Most of the expressions are expressed in Arrhenius rate form and are dependent on the surface coverage.

Hofmann et al. [58] compare the heterogeneous reaction mechanism (HCR) from [48] with a simplified approach, where the
methane reforming is described according to a global kinetic approach and the water–gas shift reaction in equilibrium. The HCR model can be used to calculate the catalytic surface coverage. It is concluded that the HCR predicts a less steep methane consumption along the flow channel. Slower methane conversion means less hydrogen available throughout the cell. Also the temperature distribution is significantly affected by the steam reforming reaction, with a flatter temperature gradient for the HCR.

4. Multiscale SOFC modeling development

4.1. Micro-, meso- and macroscale approaches

An SOFC can be described by different length scales: systemscale (~10^2 m), component scale (~10^1 m), material aspect at the fuel cell/constituent (~10^-2 m), flow/diffusion morphologies (~10^-3 m), material structure/interface (~10^-4 m, Monte Carlo (MC) and continuum methods (CM) in Fig. 7), and functional material levels (~10^-9 m, Density Functional Theory (DFT) and Molecular Dynamics (MD) in Fig. 7). Not only proper length scales are needed to describe various parts of an SOFC, also different time scales need to be considered. Cell charging and cathode gas thermal diffusion are in 10^-3 s, convective transport is in 10^-1 s, cell heating and anode streamwise thermal diffusion are in 10^3 s and cathode streamwise thermal diffusion is in 10^5 s [8]. A general relation between time- and length scales with proper modeling methods can be seen in Fig. 7. DFT and MD corresponds to the microscale, MC and CM to the mesoscale and methods with a length scales bigger than 10 μm (for example finite element method (FEM) and finite volume method (FVM)) to the macroscale. There are two basic approaches for interaction between the different modeling methods (i.e., multiscale modeling): (1) hierarchical methods and (2) hybrid or concurrent methods [64]. These approaches are further described in Section 4.1.4.

Research of the physical phenomena is based on different levels of scales: micro-, meso- and macroscales. The microscale model corresponds in many cases to the atom or molecular level as thermo- or fluid dynamics and detailed chemical reactions are studied. The microscale does not need to be as small as the size of the molecules. A mesoscale model corresponds to a larger scale than a particle but a smaller one than the facility or the global flow field. Macroscale models match to the global flow field. Microscale modeling is in general more related to theoretical knowledge compared to macroscale modeling that is more related to empirical data. Empirical parameters for macroscale models could be based on the results from micro- or mesoscale models [12].

Instantaneous flow around individual moving particles can be calculated in microscale models. Flows corresponding to calculation cells, larger than particles but smaller than global flow fields, are calculated with mesoscale models. Trajectories of individual particles are calculated with particle motion equations for microscale modeling. The flow field is in mesoscale modeling divided into a number of small cells, but not as small as the particle size [12].

Different methods have been developed to describe different scales. Methods that can be used for SOFC modeling are listed in Table 3, based on micro-, meso- and macroscales.

4.1.1. SOFC modeling at microscale

Frayret et al. [65] simulated microscopic aspects of oxygen diffusion in ceria-based materials in the ionic conductor with the DFT. This methodology is a good tool to study the connection between dopant ionic radius and diffusion at the atomic scale. DFT is an "ab initio" method where the material properties are described by solutions of the Schrödinger equation. DFT models have a characteristic length scale of Å–nm and a time scale of ps–ns.

Cheng et al. [66] simulated the oxygen ion-hopping phenomenon inside a YSZ electrolyte with MD. MD can be used to model grain boundary structure, specific heat capacity and molecular structure. Systems up to 10^5 atoms and a time scale of the order of ten ns can be modeled [67].

The Lattice-Boltzmann method (LBM) is used to model mass transport of gases inside the porous anode of an SOFC. The porous structure is based on scanning electron microscope (SEM) images, which are converted to digital form. Advantages of the LBM model...
are that a detailed analysis of mass transfer can be carried out for the actual anode microstructure; this means that tortuosity is not used as a fitting parameter. LBM approach is accurate enough to model concentration polarization in 2D. By changing the number of void spaces present in the solid matrix the porosity in the LBM is varied.

More detailed information is needed about the kinetic, thermodynamic, and transport data of species and reactions, particular for the YSZ surface but also for other parts of the cell. This is achievable either with experiments from the surface science or with microscale theoretical approaches, such as quantum chemical calculations with DFT and Molecular Dynamics calculations [44].

4.1.2. SOFC modeling at mesoscale

Modak and Lusk [67] applied kinetic MC to simulate the open-circuit voltage and electrical double layers of a doped electrolyte. Discrete time increments of various size are used to capture diffusion or adsorption in a single step. The physical property generated by QC and MD can be utilized in the KMC model. Monte Carlo methods have a characteristic length scale of 100 nm–μm and a time scale of ms–s [15].

Huang et al. [69] employed COMSOL Multiphysics to model the multiphysics processes in the SOFC cathode–electrolyte interface considering the geometry and detailed distribution of the pores and the ionic conducting phase. The charge transfer rate, electron- and ion conduction are governed in a modeling domain extracted from actual materials encountered in the application. Note that FEM is developed for phenomena occurring in the macroscale. However Huang and coauthors are solving cathode electrochemical processes at mesoscale with COMSOL Multiphysics.

4.1.3. SOFC modeling at macroscale

Cheng et al. [66] used the FEM and the commercial software COMSOL Multiphysics to solve the flow equations for macroscopic transport phenomena. Navier–Stokes equations are used to describe the flow conditions in the air and fuel channels and the Darcy law describes the flow conditions in the porous layer. FEM has a characteristic time scale of 1 s and above [65].

A clear relationship between the underlying physical conditions and numerical algorithm has made the FVM a popular method for commercial codes such as PHOENICS, FLUENT, CFX and STAR-CD [36]. Pasaogullari and Wang [70] as well as Autessier et al. [71] used FLUENT to solve the equations of momentum, mass, energy, multi-component species and electrochemical kinetics for an SOFC. Hussain et al. [17] employed the FVM to model the transport of multi-component species inside porous SOFC anodes.

Continuum electrochemical models are commonly used in many modeling studies, where the electrical current density, the cell voltage and the heat production are considered. The output voltage is normally expressed as the difference of the open-circuit voltage and the polarizations due to the electrical current generation. Continuum electrochemistry models can be used to determine the actual anode microstructure; this means that tortuosity is not used as a fitting parameter. LBM approach is accurate enough to model concentration polarization in 2D. By changing the number of void spaces present in the solid matrix the porosity in the LBM is varied.

More detailed information is needed about the kinetic, thermodynamic, and transport data of species and reactions, particular for the YSZ surface but also for other parts of the cell. This is achievable either with experiments from the surface science or with microscale theoretical approaches, such as quantum chemical calculations with DFT and Molecular Dynamics calculations [44].

4.1.4. SOFC modeling integration issues

Multi-physical problems can often be described with a set of partial differential equations at different length scales. The coupled partial differential equations can be solved simultaneously in physical domains for corresponding physical phenomena. Fuel cell operation depends on complex interaction between multi-physics such as multi-phase fluid flow, mass transport, heat transfer and (electro-) chemical reactions [73]. Two basic integration approaches can be found: hierarchical method, and hybrid and concurrent method. The hierarchical modeling starts at higher resolution (smaller scale) and properties are extracted and used as input to the next level method. The hierarchical methods are today the most developed methods for multiscale modeling [65]. Three different methods are used for hybrid approaches to describe various regions of the material with the appropriate time and length scale resolutions. The hybrid methods that permit concurrent simulations are promising for the future development, since only one calculation needs to be performed, however, it requires more computational power compared to the hierarchical methods [65].

As an example a general hybrid molecular-continuum strategy, i.e., the point-wise coupling methodology (PWC) was presented in [74]. The solution in the entire domain is advanced through the continuum solver and the atomistic models are utilized to: (1) calculate transport properties (such as viscosity for the non-Newtonian fluids and thermal conductivity), (2) provide accurate boundary condition (such as temperature, slip velocity and tangential stress), and (3) substitute constitutive relations for the pressure among others. The MD solver is occupied to calculate macroscopic quantities at selected points in time and space for the prevailing continuum conditions. The transport of information from the molecular to the continuum model is less complicated compared to the opposite direction, however it is crucial for the accuracy and efficiency of the hybrid scheme. The calculation of the macroscopic variables are performed through averaging the corresponding microscopic properties. The PWC effectively avoids performing MC simulations for nearly identical continuum states, meaning a significant reduction in computational time [74].

The particle size in SOFCs is in the sub-micron scale, and the TPBs are in nanoscale. The morphology and properties of these scales are important for the performance of the fuel cell, since they control how much of the Gibbs free energy being available. It means that the sciences at nanoscale are critical to the performance at a system-scale. A robust design and multiscale analysis should consider those nano-details as well as macro system level [75].

Khaleel et al. [72] developed a multiscale approach, where the macroscale electrochemistry model (Lattice-Boltzmann algorithm) calculates the performance of the porous electrode material based on material structure at the microstructure level, distribution of reaction surfaces, and transport of oxygen ions through the mate-
rial. The microscale electrochemistry modeling is used to calculate the overall fuel cell current–voltage relation, which is then used as input to the macroscale calculations (MARC software), where the current density, cell voltage and heat production are calculated.

Bove et al. [76] established a macroscale black-box model, using equations that are generally used for microscale models. The main problem with this approach is that the gas composition variation along the flow direction is neglected, the effect of fuel utilization variation cannot be estimated and the cell voltage is underestimated. The advantages is that the calculations are less time consuming compared to microscale modeling, and contains more information describing microscale phenomena compared to other macroscale models.

Cheng et al. [66] and Pasaogullari and Wang [70] introduced multiscale-concepts; however, they did not mention how the different scales interact with each other. It is frequently stated in the literature that data or property constants can be obtained from smaller scale and then used in a model made at a larger scale. However, information about the construction of this coupling is rare [27,64–65].

4.2. Validation

The numerical results of a computation are only an approximation of the real world conditions and considering that convergence is not sufficient, validation is a necessary step in the development of computational models [25,77]. A range of validity can be established between the developed model and experimental data. Models are usually validated by adjusting estimated physical properties values to reach certain agreement [78]. For example adjusting the reaction orders of methane and water in the methane steam reforming reaction according to experimental data is very common.

Microscale approaches, such as Lattice-Boltzmann (LB) method can be used to model multi-component gas transport in SOFCs. The diffusive flow in the porous electrodes can then be simulated without any empirical modification of diffusion coefficients, such as medium porosity and tortuosity. Validation of these microscale models are not required [79].

Tortuosity is an important parameter for characterization of fluid flow through porous media in many macroscale models. It is normally considered as a geometric parameter, however, it was originally introduced as a kinematic property, equal to the relative average length of the flow path of a fluid particle from one side of a porous structure to the other side. If a suitable model is developed for the porous medium, then the tortuosity becomes a geometric property. Frequently in the literature tortuosity is treated as a fitting property (used for validation purposes), i.e., the tortuosity should then not be seen as a kinematic or geometric property [46].

Nicotella et al. [80] has developed a model considering mass and charge transport and electrochemical reactions in porous composite cathodes. A validation is performed to compare the experimental data on polarization resistance obtained by impedance spectroscopy on cathodes of different thicknesses with the simulations. It is concluded that the developed model can replicate the experimental results concerning the dependence of the polarization resistance on the cathode thicknesses, because non-uniform morphological properties are taken into account.

Peksen et al. [81] has developed a computational model (based on FVM) for SOFC pre-reformer considering fluid flow, heat transfer, chemical reacting species transport. Surface temperature measurements and species gas compositions are used for validation of the 3D model. Also experimental validation is performed using thermocouples and gas chromatography.

Suwanwarangkul et al. [19] modeled transport phenomena inside SOFC anodes. Fick’s- Stefan–Maxwell- and Dusty-Gas models were compared. Experimental data from literature were used to compare the calculated concentration over-potential for H2–H2O–Ar and CO–CO2 systems.

4.3. State-of-the-art in multiscale SOFC modeling

Mathematical models are central tools in examining and understanding effects of various operating parameters and designs, as well as promoting the SOFC development. Results from modeling can be used to select optimal operating conditions and to optimize the design. Numerical results obtained from commercial softwares show acceptable accuracy if compared with experimental data as well other modeling predictions [4].

The current state-of-the-art in SOFC modeling is to use computational fluid dynamics (CFD) to solve the transport equations and couple the solution to an electrochemical model. Some models found in the literature are compared in terms of methodology for the modeling, see Table 4. Most of the investigated models [15,17,20,22,24–26] use the finite volume method (FVM) for solving the governing equations. Most authors [15,17,22,24–26,33,82] include a parameter study to test their models, but comparison with experimental data is rare. Regarding the flow configuration co-, counter- as well as cross flow can be found in the literature. Note that it is not relevant to compare flow configurations for models at the component level.

Most authors [15,20,22,24,82] do not include material characteristics that are dependent on the temperature. However, Refs. [15,17,33] define the conductivity as temperature dependent, while the other material characteristic parameters are still defined at one temperature only. All investigated models include governing equations for mass transport, most of them [15,20,24–26,33,82] for momentum transport, some of them for energy [15,20,26,33] and ionic transport [17,22,24–26]. Pramuanjaroenkij et al. [26] included simultaneously the governing equations for mass-, energy-, momentum-, and ionic transport. Models are developed with different aims at component, cell or stack level, and calculations considering entire fuel cells system can also be found in the literature [83], however, is not reviewed in this study. Steady-state conditions are applied in a majority of the models developed so far [19].

The fuel cell is considered as a black box in zero-dimensional modeling. The principles of thermodynamics and electrochemistry are used to find output parameters such as cell efficiency, cell voltage and power output. 1D modeling can be used to model co- and counter-flow, variation of temperature and concentrations along the flow direction may be studied. There are several sections possible for a 2D cross-section, and the connection between interconnect and electrode is normally neglected in 2D modeling. 3D modeling is needed to acquire detailed knowledge of SOFC performance [42].

Most of the studies use only common SOFC material in their modeling, new studies are definitely needed to compare alternative materials, to assess the effect of material on efficiency, degradation, etc. Many thermal models consider the cell level only and assume the cell boundaries to be insulated. Stack or system level models should be developed further. Electrochemical modeling considering microstructure geometry and material properties is so far rare in the literature.

It is common to assume that the current density is uniform over the entire electrode, note that this is basically only valid as long as the gas concentration is uniform along the flow channel. The total pressure gradient inside the anode is normally assumed to be negligible. This is correct for a very short cell and when only hydrogen and carbon monoxide is used as fuel, because no net change in the number of moles occur in the gas phase [19].
4.3.1. How far can SOFC modeling reach?

There are still unresolved issues in fuel cell modeling. The exchange current density is used to calculate the activation polarization and it depends on the catalyst for the electrochemical reactions. One problem is that the data in literature is not identical or well explained. Further study of the mechanisms behind the activation polarization is important for the further development [78].

The macroscale modeling has gained a lot of success during the last years in understanding complex phenomena occurring during FC operation and it has been possible to improve the FC design. However, the microstructure of the porous electrodes is normally not included in the model. Porosity and tortuosity are, for macroscale models, normally treated as fitting parameters. The difficulty is that these parameters are used to compensate for inaccurate modeling of several phenomena. It has also been shown that different porous microstructure can be characterized with the same porosity, but different hydraulic characteristics, i.e., phenomena at the microscale are not described well by macroscale parameters. One way to overcome these limitations is to apply LBM to simulate gas flow in porous media. It should be noted that LBMs do not require pressure–velocity decoupling or solution of a large system of algebraic equations [46]. Fuel cell modeling can be significantly improved if submodels being not too expensive (in computational time) are included in a general model [78].

A grand challenge for future SOFC modeling development is the concept of an inverse coupled multi-physics modeling, where the nano-structured material design is calculated from the defined system requirements, instead of material and other engineering specifications. The activation- and diffusion related polarization losses depend strongly on the nano-structure of the cathode and anode materials, and are good examples for application of this potential approach [75].

5. Conclusions

The fuel cell is not a new invention, the principle dates back to 1838. However, the fuel cell technology is approaching the commercial phase, the potential for the future is enormous and fuel cells can be key components in a future sustainable energy system. To achieve this, the production cost must be decreased and the lifetime must be increased. One way to decrease the operating cost and also increase the lifetime is to increase the understanding of multiscale transport- and reaction mechanism within the cell. Coupling of these multiscale phenomena, i.e., multiscale modeling is promising for future fuel cell research. Micro- and macroscale physical phenomena and chemical reactions could be solved together.

Future models need to go down in scale, to specify where the electrochemical and internal reforming reaction occurs, i.e., where
heat is released. Validation with experimental data is still too rare, and the quality of the models will be increased by doing this. Detailed analysis of mass transport phenomena with electrochemical processes, velocity, temperature distributions and the microstructure of component material properties need to be integrated for a performance analysis and efficiency optimization.

A challenge faced is to further develop multiscale models for fuel cell designs. They can provide a clear understanding of operating conditions, transport and reaction phenomena at the microscale connected to, e.g., conditions in the air and fuel channels at the macroscale. It is possible to couple different physical models, for example models at the macroscale describing transport phenomena inside an anode with a macroscale model describing the entire fuel cell. Use of multiscale modeling in fuel cell research will lead to an increased power density and also to a decreased cost for development and production, and increased energy efficiency, etc.

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References


