Condensation
(Chapter 13)

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Film condensation along a vertical wall
Condensation - Nusselt’s film theory; assumptions

- The convective transport is neglected in both the liquid and vapor layers; A linear temperature distribution exists between wall and vapor conditions (pure heat conduction through the film)
- The flow field in the vapor layer is neglected; The viscous shear of the vapor on the film is negligible at $y = \delta_f$
- The temperature in the vapor layer is constant
- Isothermal surface
- The flow of condensate in the film is laminar
- Negligible subcooling
- Fluid properties are constant
Condensation – force balance

\[ \rho_f g (\delta_f - y) dx = \mu_f \frac{du_f}{dy} dx + \rho_v g (\delta_f - y) dx \]

weight of viscous shear buoyancy force
the liquid at y due to
element displaced vapor

Integrating and using the B.C. \( u_f = 0 \) at \( y = 0 \)

\[ u_f = \left( \frac{\rho_f - \rho_v}{\mu_f} \right) g \left( \delta_f y - \frac{1}{2} y^2 \right) \]

Velocity profile in the liquid film, largest at \( y = \delta_f \)
Nusselt’s film theory

The mass flow of condensate through any x position of the film:

\[ \text{Mass flow} = \dot{m} = \int_{0}^{\delta_f} \rho_f u_f \, dy = \frac{\rho_f (\rho_f - \rho_v) g \delta_f^3}{3 \mu} \]

The heat transfer at the wall per unit depth in the area \( dx \) is:

\[ q = -\lambda_f \frac{\partial t}{\partial y} \bigg|_{y=0} \, dx = \lambda_f \frac{t_s - t_w}{\delta_f} \, dx = h_{fg} \frac{\dot{m}}{dx} \]

\[ \frac{d \dot{m}}{dx} = \frac{d}{dx} \left[ \frac{\rho_f (\rho_f - \rho_v) g \delta_f^3}{3 \mu} \right] \, dx = \frac{d}{d\delta_f} \left[ \frac{\rho_f (\rho_f - \rho_v) g \delta_f^3}{3 \mu} \right] \frac{d \delta_f}{dx} \, dx = \frac{\rho_f (\rho_f - \rho_v) g \delta_f^2}{\mu} \, \delta_f \]

\[ \text{LUND University} \]
Nusselt’s film theory

\[ \lambda_f \frac{t_s - t_w}{\delta_f} dx = h_{fg} \frac{\rho_f (\rho_f - \rho_v) g \delta_f^2}{\mu} d\delta_f \]

Integrating and using the B.C. \( \delta_f = 0 \) at \( x = 0 \)

\[ \delta_f = \left( \frac{4 \mu_f \lambda_f (t_s - t_w)x}{h_{fg} g \rho_f^2} \right)^{1/4} \]
Nusselt’s film theory

\[ \alpha = \frac{\lambda_f}{\delta_f} \]

\[ \alpha = \left( \frac{h_{fg} \rho_f^2 \lambda_f^3}{4\mu_f (t_s - t_w) x} \right)^{1/4} \]

\[ \bar{\alpha} = \frac{1}{L} \int_0^L \alpha \, dx = \frac{4}{3} \alpha_{x=L} \]
Nusselt’s film theory cont.

\[ \bar{\alpha} = 0.943 \left( \frac{h_{fg} g \rho_f^2 \lambda_f^3}{\mu_f (t_s - t_w) L} \right)^{1/4} \]

\[ \text{Re}_{\delta_f} = \frac{u_m \delta_f}{v_f} = \frac{\rho_f u_m \delta_f}{\mu_f} = \frac{\dot{m}}{\mu_f} \]

\[ \text{Re}_{\delta_f} = \frac{1}{3} \frac{\rho_f^2 g}{\mu_f^2} \left( \frac{4 \mu_f \lambda_f (t_s - t_w) x}{h_{fg} g \rho_f^2} \right)^{3/4} = 0.943 \left( \frac{\rho_f^{2/3} g^{1/3} \lambda_f (t_s - t_w) x}{h_{fg} \mu_f^{5/3}} \right)^{3/4} \]

Nusselt’s film theory valid to \( \text{Re}_{\delta_f} \sim 6 \)
Condensation - empirical correlations

\[ \bar{\alpha}_L = \frac{\lambda_f}{(v_f^2 / g)^{1/3}} \left( 1.47 \text{Re}_{\delta_f}^{0.22} - \frac{1.3}{\text{Re}_{\delta_f}} \right)^{-1} \]

\[ 6 < \text{Re}_{\delta_f} \leq 450 \]

Turbulent flow

\[ \bar{\alpha}_L = \frac{\lambda_f}{(v_f^2 / g)^{1/3}} \left( \frac{\text{Re}_{\delta_f}}{(2188 + 41(\text{Re}_{\delta_f}^{0.75} - 89.5) / \text{Pr}_f^{0.5})} \right) \]

\[ \text{Re}_{\delta_f} > 450 \]

\[ \bar{\alpha} = 0.0077 \text{Re}_{\delta_f}^{0.4} \left( \frac{\lambda_f \rho_f^2 g}{\mu_f^2} \right)^{1/3} \]
Condensation – alternate Reynolds number

\[ \text{Re}'_{\delta f} = \frac{4 \dot{m}_{\text{tot}}}{\mu_f P} \]

\[ P = \begin{cases} 
    b & \text{width for a vertical plate} \\
    \pi D & \text{for a vertical pipe}
\end{cases} \]
Condensation – improvements of Nusselt’s film theory

\[ h'_{fg} = h_{fg} (1 + 0.68 \text{Ja}) \]

\[ \text{Ja} = \frac{c_{pf} (t_s - t_w)}{h_{fg}} \]

due to subcooling of the liquid layer

The constant 0.943 is replaced by 1.13

\[ \bar{\alpha} = 1.13 \left( \frac{h'_{fg} g \rho_f^2 \lambda_f^3}{\mu_f (t_s - t_w)L} \right)^{1/4} \]
Condensation – Inclined plate

\[ g' = g \sin \theta \]

Applicable range for finite plate?
Condensation on the outside surface of tubes (pipes)

\[ \alpha = 0.728 \left( \frac{h_{fg}' g \rho_f^2 \lambda_f^3}{\mu_f (t_s - t_w) D} \right)^{1/4} \]

\[ \rho_f (\rho_f - \rho_v) \approx \rho_f^2 \]

Latent heat of vaporization evaluated at \( t_s \)
Condensation inside tubes (pipes)

low flow velocities

\[ \text{Re}_{v,i} = \left( \frac{\rho_v u_{m_v} D}{\mu_v} \right)_{i} < 35 \times 10^3 \]

\[ \bar{\alpha}_D = 0.555 \left( \frac{h'_{fg} g \rho_f (\rho_f - \rho_v) \lambda_f^3}{\mu_f (t_s - t_w) D} \right)^{1/4} \]

If \( t_w \) is unkown?
Condensation-inside horizontal tubes (pipes)

\[ \bar{\text{Nu}} = \frac{\bar{\alpha} D}{\lambda_f} = 0.026 \text{ Pr}_f^{1/3} \left( \frac{D}{\mu_f} \right) \left( \frac{G_v (\rho_f / \rho_v)^{1/2} + G_f}{G_v (\rho_f / \rho_v)^{1/2}} \right)^{0.8} \]

\[ G_v = \frac{\dot{m}_v}{A} \quad G_f = \frac{\dot{m}_f}{A} \]
Dropwise condensation

By adding chemicals to the vapor
Treat the surface with appropriate chemicals
Apply a teflon layer on the surface
Apply a thin layer of gold or any other noble metal on the surface (controversy?)

The mechanism is still a mystery.
Significant progress recently
Condensation

A general problem is the presence of non-condensable gases. These introduce additional thermal resistance which reduces the heat transfer coefficient.

As vapor condenses, the noncondensable gases are accumulated at the surface and the vapor has to diffuse across the vapor-gas mixture.
Heat transfer coefficient: Order of magnitude

- Order of magnitude for water and air at different heat transfer modes

- Balance thermal resistance in heat exchangers
Thermophysical properties

Google – NIST thermophysical properties