Evaporation (Chapter 14)

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Evaporation, Boiling

1) Local boiling or subcooled boiling
2) Boiling with net evaporation

- Pool boiling
- Forced convective boiling
Evaporation – Nukiyama’s experiment – boiling curve

Heat flux increases/decreases
Evaporation – Boiling Curve

Heat flux, $[10^5 \text{ W/m}^2]$ vs. $\Delta t = t_{\text{wall}} - t_s$

1. **Natural convection**
2. **Nucleate boiling**
3. **Transition regime**
4. **Film boiling**
5. **Saturated water on a plane surface at $p = 1$ bar**

- $q_{\text{min}}$
- $q_{\text{max}}$

Wall temperature increases/decreases
Evaporation

Principal sketch of nucleate boiling ((a) and (b)), and film boiling (c).
Bubble nucleation, growth and departure

\[ R_2 > R_1 \]
\[ R_3 < R_2 \]
\[ R_c < R_3 \]
\[ R_c < R_4 \]
Nucleate boiling heat transfer mechanisms

- **Micro-convection**
  - Diagram showing bubble motion in a liquid.

- **Transient conduction**
  - Diagram showing heat transfer through a liquid.

- **Microlayer evaporation**
  - Diagram showing evaporation from a microlayer.

Nucleate Boiling; theory, empiricism

\[ \text{Nu} = \text{function}(\text{Re, Pr}) \]

\[ u_f = \frac{q_w}{\rho_f h_{fg}} \]

\[ \text{Re} = \frac{\rho_f u_f L_k}{\mu_f} \]

\[ \text{Nu} = \frac{\alpha L_k}{\lambda_f} \]

\[ L_k = \left( \frac{\sigma}{g(\rho_f - \rho_g)} \right)^{1/2} \]

\[ \text{Nu} = \frac{1}{C_{sf}} \text{Re}^{1-n} \text{Pr}_f^m \]

\[ \alpha \] is related to bubble departure diameter
Nucleate Boiling – Rohsenow’s formula

\[
\frac{c_{pf} (t_w - t_s)}{h_{fg} \Pr_f^s} = C_{sl} \left[ \frac{q_w}{\mu_f h_{fg}} \left( \frac{\sigma}{g(\rho_f - \rho_g)} \right)^{1/2} \right]^{0.33}
\]

\[n = 1/3, \ 1+m = s\]

Eq. (14-9)

\[\sigma\ \text{is the surface tension, } \sigma = c_1 + c_2 t\]
Evaporation; Nucleate Boiling Regime

\[ \frac{q}{\mu_f h_{fg}} \sqrt{\frac{\sigma}{g \left( \rho_f - \rho_g \right)}} \]

\[ \frac{c_{pf} \left( t_w - t_s \right)}{h_{fg} \text{Pr}_f} \]
Evaporation- $C_{sl}$ and $s$ in Rohsenow’s equation

<table>
<thead>
<tr>
<th>Surface – liquid</th>
<th>$C_{sl}$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickel – water</td>
<td>0.006</td>
<td>1.0</td>
</tr>
<tr>
<td>Platinum – water</td>
<td>0.013</td>
<td>1.0</td>
</tr>
<tr>
<td>Copper – water</td>
<td>0.013</td>
<td>1.0</td>
</tr>
<tr>
<td>Brass – water</td>
<td>0.006</td>
<td>1.0</td>
</tr>
<tr>
<td>Chrome – benzene</td>
<td>0.010</td>
<td>1.7</td>
</tr>
<tr>
<td>Chrome – ethanol</td>
<td>0.0027</td>
<td>1.7</td>
</tr>
</tbody>
</table>
Evaporation, nucleate boiling: Cooper’s formula

\[ \alpha = A \cdot P_R^{(0.12-0.4343 \ln R_p)} \cdot (-0.4343 \ln P_R)^{-0.55} \cdot M^{-0.5} \cdot q_w^{0.67} \]

\( A = 55 \)

\( P_R = p / p_{\text{critical}} \) is the reduced pressure

Physical properties could be expressed as functions of the reduced pressure

M is the molecular weight \( R_p \) is the surface roughness in \( \mu \text{m} \)
Evaporation, nucleate boiling; Gorenflo’s method

\[ \alpha = \alpha_0 \cdot F_{PF} \cdot \left( \frac{q_w}{q_{w0}} \right)^b \cdot \left( \frac{R_p}{R_{p0}} \right)^{0.133} \]  \hspace{1cm} (14-12)

\[ F_{PF} = 1.2P_R^{0.27} + 2.5P_R + \frac{P_R}{1 - P_R} \]

\[ b = 0.9 - 0.3P_R^{0.3} \]

\( \alpha_0 \) valid a certain reference state, namely

\[ P_{R0} = 0.1 \quad R_{p0} = 0.4 \text{ \SI{}{\mu m},} \quad q_{w0} = 2 \times 10^4 \text{ W/m}^2 \]

Reference values for \( \alpha_0 \) in Table 14-III.
Evaporation, nucleate boiling; Gorenflo’s method – for water

\[ F_{PF} = 1.73P_R^{0.27} + \left( 6.1 + \frac{0.68}{1 - P_R} \right)P_R^2 \]

\[ b = 0.9 - 0.3P_R^{0.15} \]
Evaporation-temperature distribution in liquid phase for pool boiling

Bubble departure diameter in mm scale
Large temperature drop and high heat transfer coefficient near the wall
Evaporation: equilibrium-force balance

\[ \pi r^2 (p_{\text{bubble}} - p_{\text{liquid}}) = 2\pi r \sigma \]

\[ r = \frac{2\sigma}{(p_{\text{bubble}} - p_{\text{liquid}})} \]
Surface tension

A metal paperclip floating on water

Lotus effect
Evaporation – effects on the boiling curve

- **Subcooling**
  
  A liquid enclosed in a heated container will not stay a temperature below the saturation temperature very long. Before the liquid reaches the saturation temperature or if the warm liquid is continuously replaced by cold liquid (e.g., by forced flow) the subcooling will affect the boiling curve. It has been found that the nucleate boiling regime is not very much affected but the values of $q_{\text{max}}$ and $q_{\text{min}}$ increase linearly with the subcooling. The influence on the transition regime is less known.

- **Gravity**
  
  The influence of the gravity or other body forces is of interest as the boiling process also appears in rotating or accelerated systems. Reduction of the gravity is important for boiling processes in space applications. Because the gravity acceleration $g$ is included in most expressions its role is evident.

- **Surface roughness**
  
  A heating surface might be treated in various ways to find out the importance of the surface roughness. The effect of $q_{\text{max}}$ on surface roughness is very complicated. The film boiling regime is not affected significantly by surface properties which is understandable as the liquid phase is not in direct contact with the solid surface. The nucleate boiling regime is however affected by the surface roughness.
Evaporation – transition regime
Evaporation – Taylor instability

An instability of an interface between two fluids of different densities and with the denser fluid at the top.

\[ \lambda_T = \text{function } (\sigma, g(\rho_f - \rho_g)) \]
Evaporation – Taylor instability, dimensional analysis

\[
\lambda_T = \text{const} \cdot \sigma^a \cdot g^b \cdot (\rho_f - \rho_g)^c
\]

\[ [m] = [N \cdot m^{-1}]^a \cdot [m \cdot s^{-2}]^b \cdot [kg \cdot m^{-3}]^c \]

\[
a = 1/2, \quad b = c = -1/2
\]

\[
\lambda_T = \text{constant} \sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}}
\]

\[
\text{constant} = \begin{cases} 
2\pi \sqrt{3} & \text{for one-dimensional waves} \\
2\pi \sqrt{6} & \text{for two-dimensional waves}
\end{cases}
\]
Evaporation - arrangement of vapor jets at $q_{\text{max}}$
Evaporation – Kelvin-Helmholtz instability

\[ U \]

Flag movement

\[ \frac{\lambda_{II}}{2} \]

high pressure

low pressure
Evaporation – Helmholtz instability

\[ u_g = \sqrt{\frac{2\pi\sigma}{\rho_g \lambda_H}} \]
Evaporation- estimation of $q_{\text{max}}$ for a horizontal surface

\[ q_{\text{max}} = \rho_g h_{fg} u_g \frac{A_j}{A_h} \]

\[ A_j / A_h = \frac{\pi (\lambda_{T_i} / 4)^2}{\lambda_{T_i}^2} = \frac{\pi}{16} \]

\[ q_{\text{max}} = 0.149 \rho_g^{1/2} h_{fg} \left( g (\rho_f - \rho_g) \sigma \right)^{1/4} \quad \lambda_H = \lambda_{T_i} \quad (14-25) \]

\[ q_{\text{max}} = 0.131 \rho_g^{1/2} h_{fg} \left( g (\rho_f - \rho_g) \sigma \right)^{1/4} \quad (14-26) \]
Evaporation-other geometries, pool boiling

\[ q_{\text{max}} = \text{function}(\rho_g, \rho_f - \rho_g, g, \sigma, L, h_{fg}) \]

\[
Y_1 = \frac{q_{\text{max}}}{q_{\text{max}z}} \quad \quad \quad Y_2 = \frac{L}{\sigma} \sqrt{\frac{\rho_f - \rho_g}{g}}
\]

see Table 14-IV
Forced convective boiling for immersed bodies

\[
\frac{q_{\text{max}}}{\rho_g h_{fg} U_{\infty}} = \text{function}(\text{We}_L, \rho_f / \rho_g)
\]

\[
\text{We}_L = \frac{\rho_g U_{\infty}^2 L}{\sigma}
\]
Forced convective boiling for immersed bodies

Circular cylinder in cross flow

Low velocities

\[
\frac{q_{\text{max}}}{\rho_g h_{fg} U_\infty} = \frac{1}{\pi} \left[ 1 + \left( \frac{4}{\text{We}_D} \right)^{1/3} \right]
\]  (14-34)

High velocities

\[
\frac{q_{\text{max}}}{\rho_g h_{fg} U_\infty} = \frac{\left( \rho_f / \rho_g \right)^{3/4}}{169\pi} + \frac{\left( \rho_f / \rho_g \right)^{1/2}}{19.2\pi \text{We}_D^{1/3}}
\]  (14-35)
Forced convective boiling for immersed bodies

\[
\frac{q_{\text{max}}}{\rho_g h_{fg} U_{\infty}} \begin{cases} 
< \frac{0.275}{\pi} \left( \frac{\rho_f}{\rho_g} \right)^{1/2} + 1 & \text{high velocity} \\
> \frac{0.275}{\pi} \left( \frac{\rho_f}{\rho_g} \right)^{1/2} + 1 & \text{low velocity}
\end{cases}
\]
Boiling in tubes, flow regimes - horizontal tubes

- Bubbly
- Plug
- Stratified
- Wavy
- Slug
- Annular
- Annular with liquid spray
Boiling in tubes, flow regimes-vertical tubes

(a) homogeneous bubbles
(b) inhomogeneous bubbles
(c) slugs of the gas phase
(d), (e) partial annular flow
(f) annular flow
(g) annular flow with liquid droplets in the gas phase
Boiling in tubes, flow pattern map - horizontal tubes – Baker plot

\[ \lambda = \left( \frac{\rho_g \cdot \rho_f}{\rho_{\text{air}} \cdot \rho_{\text{H}_2\text{O}}} \right)^{1/2} \]

\[ \psi = \frac{\sigma_{\text{H}_2\text{O}}}{\sigma} \left( \frac{\mu_f}{\mu_{\text{H}_2\text{O}}} \left( \frac{\rho_{\text{H}_2\text{O}}}{\rho_f} \right)^2 \right)^{1/3} \]

\[ G_g = \frac{m_g}{A_{\text{cross}}} \]

\[ G_f = \frac{m_f}{A_{\text{cross}}} \]
Boiling in tubes, flow pattern map – vertical tubes – Hewitt and Roberts

\[ G_g = \frac{m_g}{A_{\text{cross}}} \]
\[ G_f = \frac{m_f}{A_{\text{cross}}} \]

- \( G_g^2/\rho_g \) [kg/(s^2/m)]
- \( G_f^2/\rho_f \) [kg/(ms^2)]

- Annular flow
- Annular flow with liquid droplets
- Partial annular flow
- Bubbly flow
- Slug flow

Lund University
Two-phase flow, definitions and relations

$$\varepsilon = \frac{V_g}{V_g + V_f} = \frac{V_g}{V}$$  \hspace{2cm} \text{Void fraction}

$$X_F = \frac{\dot{m}_g}{\dot{m}_g + \dot{m}_f} = \frac{\dot{m}_g}{\dot{m}}$$  \hspace{2cm} \text{flowing mass quality}

$$G = \frac{\dot{m}}{A}$$

$$\dot{m}_g = GAX_F$$

$$\dot{m}_f = GA(1 - X_F)$$

$$u_R = \frac{u_g}{u_f}$$  \hspace{2cm} \text{phase velocity ratio}
Two-phase flow, definitions and relations

\[ u_{FS} = \frac{\dot{m}_f}{\rho_f A} = \frac{G(1 - X_F)}{\rho_f} \]

Superficial velocities

\[ u_{GS} = \frac{\dot{m}_g}{\rho_g A} = \frac{G X_F}{\rho_g} \]

\[ \frac{u_g}{u_f} = \frac{\rho_f (1 - \varepsilon) X_F}{\rho_g \varepsilon (1 - X_F)} \]

(14-45)

\[ u_{FS} = u_{GS} \frac{(1 - X_F)}{X_F} \frac{\rho_g}{\rho_f} \]

(14-47)
Pressure drop for two-phase flows: Lockhart-Martinelli method

\[ \text{Re}_f = \frac{u_{FS}D}{v_f} = \frac{G(1 - X_F)D}{\mu_f} \]

\[ \Delta p_f = \int_0^L f_f \frac{1}{D} \frac{\rho_f u_{FS}^2}{2} \, dz \]

\[ \text{Re}_g = \frac{u_{GS}D}{v_g} = \frac{GX_F D}{\mu_g} \]

\[ \Delta p_g = \int_0^L f_g \frac{1}{D} \frac{\rho_g u_{GS}^2}{2} \, dz \]

Index | Liquid | Gas
--- | --- | ---
 t-t | Turbulent | Turbulent
 v-t | Laminar | Turbulent
 t-v | Turbulent | Laminar
 v-v | Laminar | Laminar
Pressure drop for two-phase flows: Lockhart-Martinelli method

\[ X^2 = \frac{(dp / dx)_f}{(dp / dx)_g} \]

Martinelli parameter

\[ \phi_f^2 = \frac{(dp / dx)_{TF}}{(dp / dx)_f} \]

two-phase multiplier

\[ \phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \]

- \(C = 20\) if turbulent flow prevails in the liquid as well as in the gas (tt)
- \(C = 12\) if the liquid flow is viscous (laminar) and the gas flow is turbulent (vt)
- \(C = 10\) if the liquid flow is turbulent and the gas flow is laminar (tv)
- \(C = 5\) if laminar flow prevails in the liquid as well as in the gas (vv)
Pressure drop for two-phase flows: Friedel’s method

\[
\left( \frac{dp}{dx} \right)_{TF} = \phi^2_{LO} \left( \frac{dp}{dx} \right)_{LO}
\]

LO means liquid only

Formulas for \( \phi^2_{LO} \) see book.
Oil recovery from deep sea: pressure drop

- Gas-oil or gas-oil-water multiphase flow

- Temperature along the oil pipes from deep sea affects thermophysical properties, e.g., oil viscosity varies a lot with temperature

- Low flow velocity, gravitational loss dominated

- High flow velocity, frictional loss dominated
Forced convective boiling – heat transfer and temperature distribution

**TEMPERATURE PROFILE**
- **x = 1**
  - Dryout
- **x = 0**
  - Fluid temperature
  - Wall temperature
  - Saturation temperature

**FLOW TYPE**
- Single phase liquid
- Bubble flow
- Slug flow
- Annular flow
- Liquid drops in the vapor
- Annular flow liquid drops in the vapor

**HEAT TRANSFER REGIMES**
- **A** Convection to liquid
- **B,C** Subcooled boiling
- **D** Saturated nucleate boiling
- **E** Forced convection across a liquid film
- **F** Dry out
- **G** Convection to vapor
- **H**
Chen’s method for estimating the heat transfer during forced convective boiling

\[ \alpha_{TF} = S\alpha_{KK} + F\alpha_C \]

Approximative range of data points

\[ X_{tt} = \left( \frac{1 - X_F}{X_F} \right)^{0.9} \left( \frac{\rho_g}{\rho_f} \right)^{0.5} \left( \frac{\mu_f}{\mu_g} \right)^{0.1} \]

\[ X_{tt} = \sqrt{\frac{(dp/dx)_f}{(dp/dx)_g}} \]
Chen’s method for estimating the heat transfer during forced convective boiling, continued

\[ F = \left( \frac{\text{Re}_{TF}}{\text{Re}_f} \right)^{0.8} \]

\[ F = \begin{cases} 1 & \text{if } \frac{1}{X_{tt}} \leq 0.1 \\ 2.35 \left( \frac{1}{X_{tt}} + 0.213 \right)^{0.736} & \text{if } \frac{1}{X_{tt}} > 0.1 \end{cases} \]

\[ \alpha_C = 0.023 \text{Re}_f^{0.8} \text{Pr}_f^{0.4} \frac{\lambda_f}{D} \]

\[ \text{Re}_f = G(1 - X_f)D / \mu_f \]
Chen’s method for estimating the heat transfer during forced convective boiling, continued

\[ S = \frac{1}{1 + 2.53 \cdot 10^{-6} \text{Re}_{TF}^{1.17}} \]

\[ \alpha_{KK} = 0.00122 \frac{\lambda_f^{0.79} c_{pr}^{0.45} \rho_f^{0.49}}{\sigma^{0.5} \mu_f^{0.29} h_{fg}^{0.24} \rho_g^{0.24}} \Delta t_s^{0.24} \Delta p_s^{0.75} \]

\[ \Delta t_s = t_w - t_s \]

\[ \Delta p_s = p_s(t_w) - p_s(t_s) \]
Chen’s method for estimating the heat transfer during forced convective boiling, continued

Calculate $\alpha_{TF}$ for a number of $\Delta t_m$ according to

$$\alpha_{TF} = S\alpha_{KK} + F\alpha_C$$

Then create a graph $q = \alpha_{TF} \Delta t_m$ vs $\Delta t_m$

At $q = q_w$ find the true $\Delta t_m$
Alternative method for estimating the heat transfer during forced convective boiling - Gungor & Winterton

\[ \alpha_{TF} = S\alpha_{KK} + E\alpha_C \]

\[ E = 1 + 2.4 \cdot 10^4 \text{Bo}^{1.16} + 1.37(1 / X_t)^{0.86} \]

\[ \text{Bo} = \frac{q_w}{(G \cdot h_{fg})} \]

\[ S = \frac{1}{1 + 1.15 \cdot 10^{-6} E^2 \text{Re}_f^{1.17}} \]

Here \( \alpha_{KK} \) is taken from Cooper’s formula, \( \alpha_C \) from Dittus-Boelter’s equation.
Alternative method for estimating the heat transfer during forced convective boiling - Steiner & Taborek

\[ \alpha_{TF} = \left( \alpha_{KK}^n + \alpha_C^n \right)^{1/n} \]

Here \( \alpha_{KK} \) is taken from Gorenflo’s method, \( \alpha_C \) from Gnielinski’s formula

\( n = 3 \)
Thank you very much!!