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Heat Transfer in Gas Turbine Combustors

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ABSTRACT
The heat transfer in a gas turbine combustor can be separated into convective, radiative and conductive heat transfer. As far as the heat transfer to and from the exposed liner wall is considered, the conductive part is negligible in relation to the convective and radiative heat transfer. Due to the thermal limit of the liner wall material, it is of interest to predict the wall temperatures at the liner wall. This can be done either by empirical correlations which will give an average temperature, or by numerical methods which will give a more detailed temperature distribution. In the numerical approach one has to model the chemical reactions due to the combustion and also chose an appropriate turbulence model due to the complex flow.

NOMENCLATURE

C/H	Carbon/hydrogen mass ratio of fuel
D_L	Liner diameter (can) or height (annular), m
D_h	Hydraulic mean diameter, m
G	Incident radiation, W/m^2
h	Heat-transfer coefficient, $W/(m^2K)$
h	Enthalpy, J/kg
I	Intensity, $W/(m^2sr)$
k	Thermal conductivity, W/(mK)
L	Luminosity factor
l_b	Mean beam length of radiation path, m
P	Total pressure, kPa
q	Fuel/air ratio by mass
Re	Reynolds number
s	Slot height, m
S_{ij}	Strain rate tensor, s^{-1}
U	Flow velocity, m/s
V_i	Mass diffusion velocity vector, m/s
x	Distance downstream of slot, m
Y_k	Mass fraction of species k
α	Absorptivity
δ_{ij}	Kronecker delta
ε	Emissivity
κ	Absorption coefficient, m^{-1}
μ	Dynamic viscosity, kg/(ms)
ρ	Density, kg/m^3
σ	Stefan-Boltzmann constant, $W/(m^2K^4)$
ω_k	Formation rate, $kg/(m^3s)$

INTRODUCTION
Gas turbines are widely used in aircrafts and in industrial processes due to the relative high specific work output. Gas turbines are also commonly used as back-up and primary power generators due to the fast ramp-up speed of a gas turbine. The specific power output of the gas turbine is mainly limited by the pressure rise that the compressor is able to deliver and the temperature rise in the combustor. With the ambition to increase the combustion temperature to achieve higher power output, there is a need to predict the temperature in the exposed materials in the combustor and to identify where possible hot-spots can occur.

PROBLEM STATEMENT
Figure 1 shows a combustor and the main airflows within. Fuel is supplied through the fuel injector and initially mixed together with only a part of the inlet airflow. Some of the cold airflow is supplied through the liner wall continuous along the combustor length to achieve a satisfying combustion and to cool the liner wall. The rest of the cold airflow, if no air for blade cooling is needed, is mixed together with the hot gas at the outlet of the combustor to achieve an acceptable temperature profile which fulfills the demands for the turbine inlet.

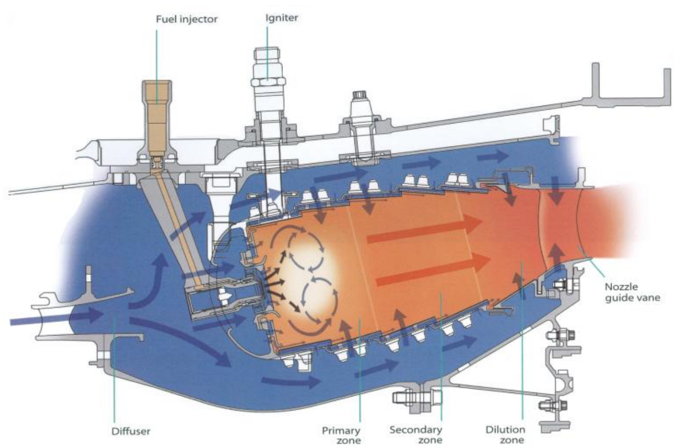

Figure 1 **(Genrup, 2012)**

Figure 2 shows components of the heat flux in an element of the liner wall (1).

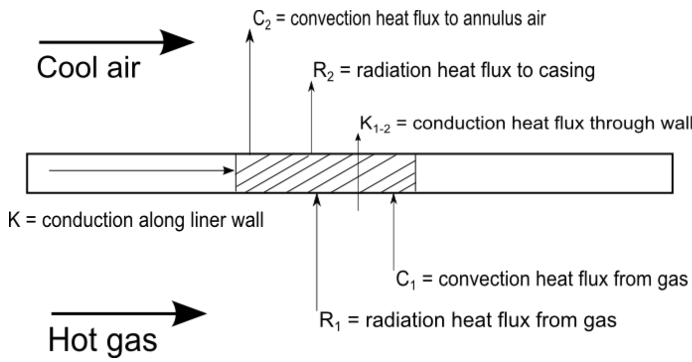

Figure 2

With direction of the fluxes as in Figure 2 and with the assumption that the liner wall is thin ($A_{inner} \approx A_{outer}$), heat balance of the marked element can be written as

$$R_1 + C_1 + K = K_{1-2} = C_2 + R_2 \qquad \text{Eq. 1}$$

The problem to determine the thermal load at the liner wall then comes down approximate these components.
There are two major approaches to evaluate the fluxes as shown in Figure 2, either by empirical correlations or by numerical methods.

LITERATURE SURVEY

The heat transfer in gas turbine combustor has been subject to many research projects and, due to the complex phenomena of chemical reactions, turbulent flow and heat transfer, still is. Lefebvre's and Ballal's (1) work address many empirical correlations to describe the heat transfer in different types of combustion. Bahador. M (Mehdi, 2007) has done a doctoral thesis on the subject with focus on how to model the radiative heat transfer. The doctoral thesis covers both empirical and numerical methods, where the empirical methods mostly are a summery of Lefebvre's work.

PROJECT DESCRIPTION

The empirical approach

As described in Lefebvre's work (1) can the contribution from the conduction K in Figure 2 always be neglected compared to the other components and Eq. 1 is then reduced to

$$R_1 + C_1 = K_{1-2} = C_2 + R_2$$

The conduction through the liner wall can be described as

$$K_{1-2} = \frac{k_w}{t_w}\,(T_{w1} - T_{w2}) \qquad \text{Eq. 2}$$

If the inner liner wall is considered as a black body the net internal radiation is (2)

$$R_1 = \varepsilon_g \sigma T_g{}^4 - \alpha_g \sigma T_{w1}{}^4 \qquad \text{Eq. 3}$$

In real application the liner wall can't be seen as a black body and Lefebvre (1) suggested that introducing the factor $0.5(1+\varepsilon_w)$ into Eq. 3 should compensate for that effect. Further, Lefebvre suggests that the ratio

$$\frac{\alpha_g}{\varepsilon_g} = \left(\frac{T_g}{T_{w1}}\right)^{1.5}$$

is a valid approximation. With these assumptions Eq. 3 can be written as

$$R_1 = 0.5(1+\varepsilon_w)\varepsilon_g T_g{}^{1.5}(T_g{}^{2.5} - T_{w1}{}^{2.5}) \qquad \text{Eq. 4}$$

According to Lefebvre (1), the heat transferred by radiation can be divided into two components, non-luminous and luminous gases. The contribution from the different components can be brought into the equation by evaluate the gas emissivity as

$$\varepsilon_g = 1 - \exp[-290PL(ql_b)^{0.5}T_g{}^{-1.5}]$$

Where P is the pressure, L the luminosity factor, q the air to fuel ratio, l_b the beam length and T_g the hot gas temperature. Lefebvre (1) presented a correlation for the luminosity factor L according to

$$L = \frac{336}{H^2}$$

where H is the hydrogen content in the fuel in mass percent.

The radiation from the outer liner wall to the casing is described as radiation between two gray bodies according to Lefebvre (1) and can, if respectively wall temperature is assumed to be constant in axial direction, be formulated as

$$R_2 A_w = \frac{\sigma(T_{w2}{}^4 - T_c{}^4)}{\frac{1-\varepsilon_w}{\varepsilon_w A_w} + \frac{1}{A_w F_{wc}} + \frac{1-\varepsilon_c}{\varepsilon_c A_c}} \qquad \text{Eq. 5}$$

Further simplifications can be made with additional assumptions and Lefebvre proposes following simplification of Eq. 5 for aluminum respectively steel casing

$$R_2 = 0.4\,\sigma(T_{w2}{}^4 - T_3{}^4)$$

and

$$R_2 = 0.6\,\sigma(T_{w2}{}^4 - T_3{}^4)$$

The heat transferred to and from the liner wall due to convection is

$$C_1 = h_1(T_g - T_{w1}) \qquad \text{Eq. 6}$$

and

$$C_2 = h_2(T_{w2} - T_a) \qquad \text{Eq. 7}$$

The challenge is to find an acceptable approximation of the heat transfers coefficients h_1 and h_2, which of course is troublesome due to the complex flow and the chemical conditions, especially in the primary zone. Lefebvre (1) makes the assumption that the convective heat transfer in a combustor has sufficiently similarities with the convective heat transfer in a pipe to be treated accordingly. The internal convection can then be described as

$$C_1 = 0.020 \frac{k_g}{D_L{}^{0.2}} \left(\frac{\dot{m}_g}{A_L \mu_g} \right)^{0.8} (T_g - T_{w1}) \qquad \text{Eq. 8}$$

where D_L is the hydraulic diameter of the liner. For the convective heat transfer in the primary zone the factor 0.020 can be set to 0.017 to compensate for a somewhat lower bulk temperature T_g. The external convection is, with the same treatment, described as

$$C_2 = 0.020 \frac{k_a}{D_{an}{}^{0.2}} \left(\frac{\dot{m}_a}{A_{an} \mu_a} \right)^{0.8} (T_{w2} - T_a) \qquad \text{Eq. 9}$$

where D_{an} is the hydraulic diameter of the annulus air space.

All these expressions can be brought back into Eq. 1 from which the wall temperatures T_{w1} and T_{w2} can be calculated. However, if the liner wall is treated with some kind of film-cooling method, as is common in most combustors today, corrections to the expressions for the internal convection C_1 have to be made. A schematic figure of the film-cooling process is shown in Figure 3.

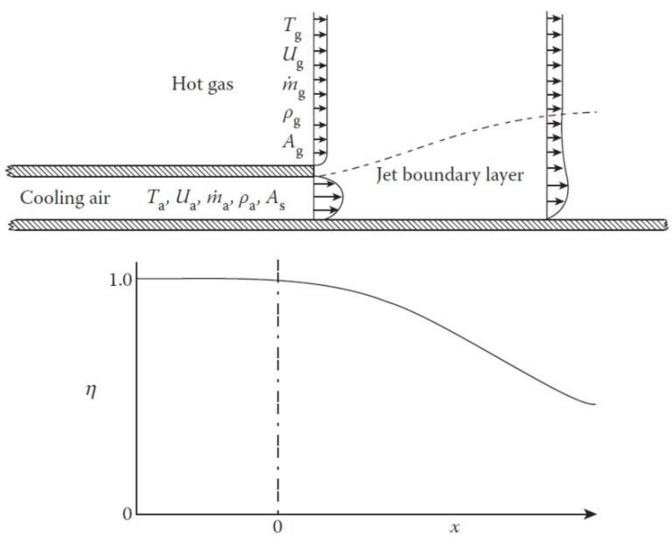

Figure 3 **(1)**

To take account for the cooling effect a film-cooling effectiveness is defined as

$$\eta = \frac{T_g - T_{w,ad}}{T_g - T_a}$$

where $T_{w,ad}$ is the adiabatic wall temperature. $T_{w,ad}$ is then used to calculate the convective heat transfer, as proposed by Lefebvre (1), accordingly to

$$C_1 = 0.069 \frac{k_a}{x} Re_x{}^{0.7} (T_{w,ad} - T_{w1}) \text{ if } 0.5 < m < 1.3$$

$$C_1 = 0.10 \frac{k_a}{x} Re_x{}^{0.8} \left(\frac{x}{s}\right)^{-0.36} (T_{w,ad} - T_{w1}) \text{ if } 0.5 < m < 1.3$$

where is $m = \frac{(U\rho)_a}{(U\rho)_g}$.

The adiabatic wall temperature is calculated from the film-cooling effectiveness. Dependent of the ratio between the cooling air stream velocity and the main stream velocity the boundary layer downstream the cooling slot is treated in two different ways, by the Turbulent boundary layer model for modest ratio or by the Wall-jet model when the cooling air stream velocity is much greater than the main stream velocity.
Expressions for the film-cooling effectiveness for the different models are further investigated by Lefebvre (1).

The numerical approach

Whereas empirical methods give a quick and initially good approximation of what average temperatures to expect in the combustor, they lack in detail and precision. With valid mathematical models of the processes that occur in the combustor, the possibility to more precisely describe the phenomena increases with the available computational power. The governing equations of interest is presented by Bahador (3) as

Continuity: $$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

Momentum: $$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

Energy: $$\frac{\partial \rho h}{\partial t} + \frac{\partial \rho u_i h}{\partial x_i} = \frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x_i} - \frac{\partial q_i}{\partial x_i} + \tau_{ij} \frac{\partial u_i}{\partial x_j} + S_q$$

Species: $$\frac{\partial \rho Y_k}{\partial t} + \frac{\partial \rho u_i Y_k}{\partial x_i} = -\frac{\partial \rho V_{i,k} Y_k}{\partial x_i} + \omega_k \quad (k = 1, .., N)$$

where

$$\tau_{ij} = 2\mu S_{ij} - \frac{2}{3} \frac{\mu \partial u_k}{\partial x_k} \delta_{ij} \qquad \text{Eq. 10}$$

and

$$q_i = -k\frac{\partial T}{\partial x_i} + \rho\sum_{k=1}^{N} h_k Y_k V_{i,k} \qquad \text{Eq. 11}$$

The heat source term in the energy equation is, with the thermal radiation conditions in a combustor, given by

$$S_q = -\nabla \hat{q}_r(\vec{r}) \qquad \text{Eq. 12}$$

Where the divergence of the radiative heat flux is derived from the radiative transfer equation (RTE) as done by Bahador (3) and can be written as

$$-\nabla \hat{q}_r(\vec{r}) = \kappa(\vec{r})(4\pi I_b(\vec{r}) - G(\vec{r})) \qquad \text{Eq. 13}$$

Further details regarding the RTE can be found in reference (3) and (4). To solve Eq. 13 in any real application one has to rely on numerical approximations. Bahador (3) suggests either the Discrete Ordinates Method (DOM) or the Finite Volume Method (FMV) but other possibilities are available such as the Spherical Harmonics Method (P_N-approximation) and the Zonal Method.

In addition to the special treatment of the radiative heat flux some model is needed to take the turbulent flow into account. Normally the Reynolds averaged Navier-Stokes (RANS) model or Large Eddy Simulation (LES) is used in the industry. For research purposes or if a fully resolved flow field is required the Direct Numerical Simulation (DNS) may be used. For more details regarding turbulence modeling the reader is referred to CFD-literature.

Another troublesome process to model is the combustion process, represented by the reaction rate ω_k in the conservation of species equation. A mean reaction rate model is presented by Bahador (3) and defined as

$$\omega_k = -\frac{\bar{\rho}A_{EDM}}{t_T}\min(\tilde{Y}_f, \frac{\tilde{Y}_o}{s_o}, B_{EDM}\frac{\tilde{Y}_P}{s_P}) \qquad \text{Eq. 14}$$

where A_{EDM} and B_{EDM} are constants and

$$s_o = \left(\frac{oxygen}{fuel}\right)_{stoich}, s_p = \left(\frac{products}{fuel}\right)_{stoich}$$

where the ratios are based by mass.

This model is suitable for both premixed and diffusion flames. Even though there are models for the reaction rate, the process of soot-formation in a turbulent flame is still rather unresolved. The formation of soot is a process which greatly influences the radiative heat flux.

When appropriate numerical treatment of the radiative heat flux and turbulence model is chosen, the local wall temperature for each point can be solved from

$$q'' = h(T_w - T_f) - q_{rad}'' \qquad \text{Eq. 155}$$

CONCLUSIONS

The empirical and numerical approach to predict the liner wall temperature both has its advantages and disadvantages and one may argue that one of them cannot fully replace the other. As far as local accuracy goes, the numerical methods gives a much better approximation, even though improvements regarding the modeling of the more complex physical and chemical processes is desirable. The drawback is of course the computational power and time required to get an acceptable result, whereas empirical correlations gives a quick, but nonetheless useful, result even by hand calculation. Whether numerical methods give better results than empirical is of course a question about the requirements of the accuracy. In an initial design stage of a combustor, empirical correlations probably provides a sufficient prediction of the thermal load whereas in later stages, if more detailed data is required, a numerical analysis provides more information.

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