Heat transfer in curved pipes

Christian Carlsson

May 2014

1 Introduction

Heat transfer in pipes has important applications in many areas, such as for e.g. heat exchangers. To improve this type of equipment, a good understanding is needed of the relation between the velocity field and the temperature field. To be able to control the rate of heat transfer between the pipe wall and the fluid, a division is made into active (e.g. inducing vibrations) and passive techniques. A popular passive technique for heat exchangers, to enhance the heat transfer rate, is to use curved pipes (e.g. helically coiled, because of their compact structure).

In this field, the aim of a large portion of studies is to investigate the heat transfer rate between the solid wall and the working fluid in the pipe, in particular focusing on the potential heat transfer enhancements caused by curvature. However, to be able to contrast the results for the curved pipes, this short survey will begin by summarizing some of the important points regarding the flow and heat transfer in straight pipes. A lot of general concepts will be introduced in the straight pipe section.

2 Straight pipes

Before curved pipes are to be tackled, a short introduction to the properties of straight pipes seem to be in order. The pipes are considered to have circular cross section.

2.1 Isothermal flow in straight pipes

Flow regions in straight pipes are often classified based on the Reynolds number $Re = \rho U_b D/\mu$, where ρ is the (mass) density, U_b is the bulk flow, D is the pipe diameter, and μ is the dynamic viscosity. Fully developed flow in straight pipes with circular cross section admit a steady state analytic solution to the Navier-Stokes equations, called (Hagen-)Poiseuille flow,

$$\vec{U}(r) = 2U_b \left(1 - \frac{r^2}{R^2}\right)\hat{z} \tag{1}$$

where r is the radial distance from the pipe centerline, $R = D/2$, and $\hat{z}$ is a unit vector along the pipe axis. The velocity field in eq. (1) is *stable* only for low Reynolds number, say $Re < 2000$, where the flow is *laminar*. For large Re the flow becomes unstable, and eventually *turbulent*. It should however be noted that the linearized Navier-Stokes equations for pipe flow are asymptotically ($t \to \infty$) stable for *all* Reynolds numbers (Re arbitrary large). This means that the pipe flow can be kept laminar for much larger Re than stated above, given that the inlet flow has a *very* low disturbance level, together with a low roughness of the wall. The primary route to turbulence for pipe flow, given that such a notion even exists, is unknown.

Flow in the entrance region, which is developing (changing in the z-direction), may have a very different profile from that in eq. (1).

2.2 Heat transfer in straight pipes

A central quantity for heat transfer between the pipe wall and the fluid (inside the pipe) is the *heat transfer coefficient* h,

$$h = \frac{q}{A(T_w - T_\infty)}$$

where q is the heat transfer from the wall to the flow (measured in Watts W), A is the surface area, T_w is the wall temperature, and T_∞ is the reference temperature of the flow. A non-dimensional parameter frequently used is the Nusselt number,

$$Nu = \frac{hL}{k} \tag{2}$$

where L is a characteristic length and k is the thermal conductivity. In a step further, $k = \alpha/(\rho c_p)$, α being the thermal diffusivity and c_p the specific heat (at constant pressure). For engineering applications, eq. (2) is typically used for finding h, in cases where *empirical* expressions for the Nusselt number exist. These empirical expressions of course depend on how the heat is transferred in the particular system under consideration, which typically is very complicated, requiring detailed numerical or experimental investigations.

However, under certain restrictive conditions, analytical expression can be obtained for the Nusselt number. For example, for the fully developed flow in eq. (1), assuming that the temperature doesn't affect the flow, the heat transfer rate for a constant wall temperature $T_w > T_\infty$ gives a constant Nusselt number, $Nu = 3.66$. Similarly, using the same assumption for a constant wall heat flux, the constant Nusselt number $Nu = 4.36$ is obtained.

A dimensionless parameter indicating the importance of buoyancy is the Rayleigh number (Ra). For flow where buoyancy is of primary importance, leading to so-called *natural convection*, the Nusselt number can be written as

$$Nu = Nu(Ra, Pr, ...)$$

Considering Rayleigh numbers below the critical, given that the flow is laminar, heat transfer perpendicular to the flow is typically dominated by conduction.

Keep in mind that when buoyancy (gravity) starts to play a role, the orientation of the pipe becomes important. The Prandtl number $Pr = \mu/(\rho\alpha)$ is usually also involved. Note that the Prandtl number is typically only weakly temperature dependent. For convective heat transfer in flow which is not induced by buoyancy, called *forced convection*, the Nusselt number instead becomes

$$Nu = Nu(Re, Pr, ...)$$

showing a Reynolds number dependence. The constant Nusselt number results stated above are examples of forced convection. Extending the result for a constant heat flux, allowing for buoyancy involving small rates of heating, was done for a *horizontal* pipe by Morton (1958). The regions of interest were considered to be far from the pipe entrance (giving fully developed profiles), and the properties of the fluid were temperature independent, except for the density in the buoyancy terms (Boussinesq approximation). The most important parameter turned out to be the product $ReRa$ of the Reynolds number and the Rayleigh number. The Rayleigh number was defined as

$$Ra = \frac{\beta\rho g\tau R^4}{\alpha\mu}$$

where g is the gravitational acceleration, β is the thermal expansion coefficient, and τ is the constant axial temperature gradient (which follows from the constant heat flux at the wall). When buoyancy is added, the colder fluid in the core moves downward and leads to two vertical vortices. The flow structure normal to the pipe axis can be seen qualitatively in fig. (1b). Also, the maximum axial velocity, which is located in the center of the pipe for the case without buoyancy, is moved downward. This enhances the heat transfer rate on the bottom part of the pipe.

Similar to the velocity field, the temperature field and heat transfer characteristics can look very different close to the pipe inlet compared to the fully developed situation, giving a thermal entrance region. Normally a flow heat exchanger is designed to be short, to take advantage of the relatively large heat transfer rates which typically appear in the thermal entrance region. However, it should be noted that much more is known in general about fully developed flow compared to developing flow.

For turbulent flow, the situation changes drastically, and the Nusselt number may increase by several orders of magnitude. This is a result of the mixing brought about by the unstable flow, and in particular the velocity fluctuations in the wall normal (radial) direction. The flow and thermal entrance regions are generally short for turbulent flow, and typically only fully developed flows are studied.

3 Curved pipes

Heat transfer in curved pipes is considered in this section, which is often used to enhance the heat transfer rate h (or Nu). The focus in this section is on pipe bends in a single plane (and with a constant curvature radius). However, a lot of work has been done for heat transfer in helically coiled pipes, involving a pitch (or helix angle), as reflected in the review article by Naphon & Wongwises (2004).

3.1 Isothermal flow in curved pipes

Flow in curved pipes, due to centrifugal forces, gives rise to secondary flow. The secondary flow structure takes the form of two counter-rotating axial vortices, called the *Dean vortices*. Furthermore, the maximum axial velocity is shifted towards the outer side of the pipe bend, giving rise to a larger shear stress at the outer wall. For small curvature ratios $\gamma \equiv R/R_c$, where R_c is the radius of curvature, the Dean number

$$De = \sqrt{\gamma} Re$$

is a similarity parameter. For a review of laminar flow in curved pipes, see Berger *et al.* (1983). The secondary motion of course not only affects (important quantities such as) the pressure drop, but also the heat transfer characteristics.

3.2 Heat transfer in curved pipes

Fully developed laminar flow in heated curved pipes, with circular cross section, were studied analytically by Yao & Berger (1978). The pipes were heated at a uniform rate, giving a constant temperature gradient along its axis, and the flow experienced both centrifugal and buoyancy forces (using the Boussinesq approximation). The buoyancy terms, as stated in the section for the straight pipe, will make the (cold) fluid in the core move downward and lead to two "vertical" vortices (when centrifugal forces are excluded). The resulting flow, including *both* centrifugal forces and buoyancy, can be considered to lead to approximate *superpositions* of the different flow modes (vortices). Both horizontal and vertical pipes were considered, where perturbation expansions were made for small values of De and $ReRa$. The results were considered to be valid for $De^2 \lesssim 500$ and $ReRa \lesssim 3000$, and arbitrary values of Pr. The Reynolds number should be considered small enough to ensure laminar flow. For the vertical 180° curved pipe ("U-bend"), the two forces may either enhance each other or suppress each other, depending on the location along the bend. At the entrance of the bend, given that the flow travels upward, the two forces point in the same direction, and the maximum axial flow is displaced towards the outer side of the bend. In the middle of the pipe the centrifugal and buoyancy forces are perpendicular, and the maximum axial velocity is again shifted towards the outer side of the bend. After the 180° bend the centrifugal and buoyancy forces act in opposite directions, and the resulting profile depends on the relative strength of the two forces. The maximum axial velocity moves towards the outer part of the bend when buoyancy is weak compared to the centrifugal force, and towards the inner part when the buoyancy dominates over the centrifugal force. In particular, if the forces are comparable in strength, the axial velocity distribution may be

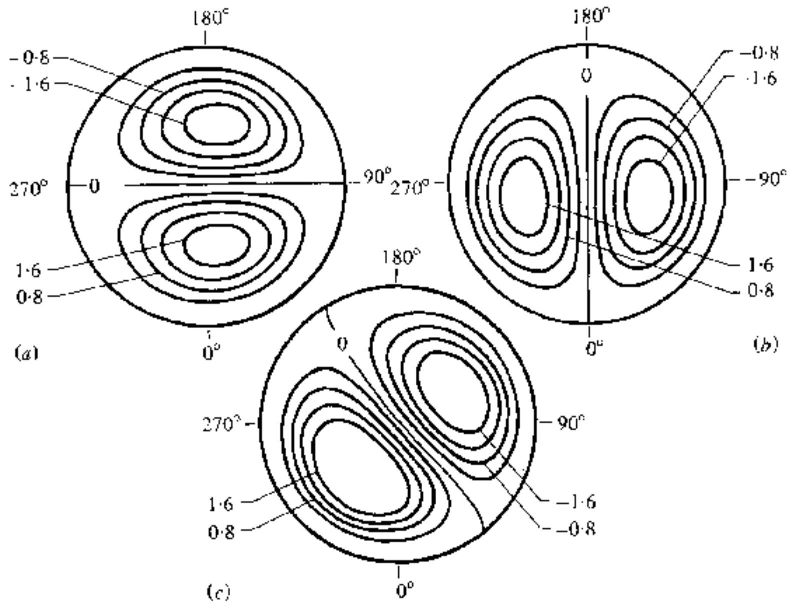

Figure 1: Streamlines, for the flow normal to the pipe axis, for a horizontal pipe. (a) $ReRa = 0$, $De \neq 0$; (b) $ReRa \neq 0$, $De = 0$; (c) $ReRa \neq 0$, $De \neq 0$. Image taken from Prusa & Yao (1982).

close to a Poiseuille profile. The temperature distribution was distorted in a similar way as the axial velocity profiles (for both the horizontal and vertical pipes). Nusselt number dependencies were also calculated analytically, which for the horizontal pipe took the form

$$Nu = Nu(Re, Ra, Pr, De, \psi)$$

where ψ is the pipe circumferential angle, while for the vertical pipe there was an additional dependence on the position ($\tilde{\theta}$) along the pipe bend.

In the article of Prusa & Yao (1982), focusing only on horizontal curved pipes, larger values of De and $ReRa$ were considered than in Yao & Berger (1978). Again, the boundary conditions at the wall gave a constant axial temperature gradient along the pipe. Results are shown in fig. (1), where $ReRa = 0$ implies no buoyancy while $De = 0$ implies no pipe curvature. The Prandtl number was constant, $Pr = 1$. For a given axial pressure gradient, the mass flow rate drastically reduced due to the secondary motion (because of the increased dissipation). More importantly, again for a given axial pressure gradient, the total heat transfer rate was seen to decrease for increasing curvature or increasing axial temperature gradient. A flow regime map was also given, showing where the two forces dominate or are comparable in ($ReRa$,De)-space.

A numerical study of a horizontal 90° curved pipe (with circular cross section) was performed by Sillekens *et al.* (1994). The flow was laminar ($Re = 500$), the curvature ratio $\gamma = 1/14$, and $Pr = 0.7$. The incoming flow had a parabolic velocity profile (see eq. (1)) and a constant temperature T_∞, while the wall in the bend had a constant temperature $T_w > T_\infty$. The Boussinesq approximation was used (and heat generation due to viscous dissipation was neglected). The secondary flow, like for the cases above, resulted from the centrifugal influence (resulting in a flow speed u_{De}) along with the buoyancy (resulting in a flow speed u_{Gr}). The ratio of the two speeds were given as

$$\frac{u_{Gr}}{u_{De}} = O\left(\frac{\sqrt{Gr}}{De}\right)$$

introducing the Grashof number Gr,

$$Gr = \frac{g\rho^2\beta(T_w - T_\infty)D^3}{\mu^2}$$

For $\sqrt{Gr}/De = 0$, forced convective heat transfer is obtained (as in fig. (1a)), while for $\sqrt{Gr}/De$ sufficiently large, the convective heat transfer is mixed (as in fig. (1c)). Velocity and temperature distributions are shown in fig. (2). The temperature field is seen to correlate closely with the velocity field. When $\sqrt{Gr}/De = 0$, the temperature distribution is seen to be symmetric about the horizontal center line, along with the Dean vortices. The Nusselt number is seen to reach a maximum value on the outer side of the pipe, and decreases significantly towards the inner side. For $\sqrt{Gr}/De > 0$, on the other hand, the Dean vortices are skewed, and the maximum axial velocity (and the maximum axial velocity gradients and temperature gradients) is moved downward along the outer side of the bend. The location for the maximum Nusselt number therefore also gradually moves down in the pipe as $\sqrt{Gr}/De$ increases. The variation of the averaged Nusselt number along the bend, together with an example of the variation between the inner and outer side, is given in fig. (3). The peak in the result for $\sqrt{Gr}/De = 4.09$, to the right in fig. (3), is due to insufficient resolution. The heat transfer characteristics are found to be greatly enhanced in the curved pipe compared to a straight pipe (up to 250%), subject to the same flow rate.

4 Summary & Conclusion

A rough overview of the flow and heat transfer characteristics in curved pipes have been given. Some of the relevant dimensionless parameters have been introduced, which can help in determining the relative importance of the different phenomena. In general, the use of curved pipes leads to enhanced heat transfer compared to straight pipes, at least for a given flow rate. Note that radiation has been neglected in all cases considered.

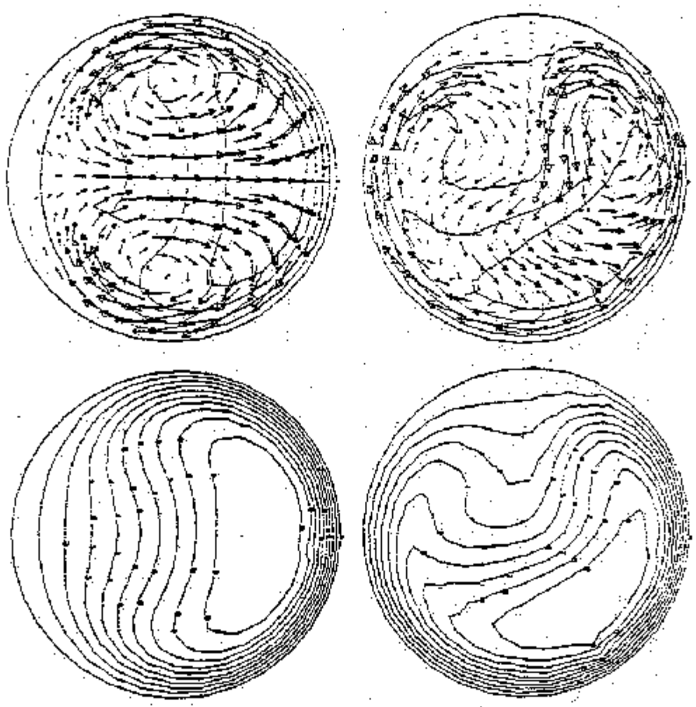

Figure 2: Cross sections of velocity and temperature distributions at 29° along the horizontal pipe bend. Top: secondary velocity vectors and contours of the axial velocity, bottom: contours of temperature. Left: $\sqrt{Gr}/De = 0$, right: $\sqrt{Gr}/De = 4.09$. Images taken from Sillekens *et al.* (1994).

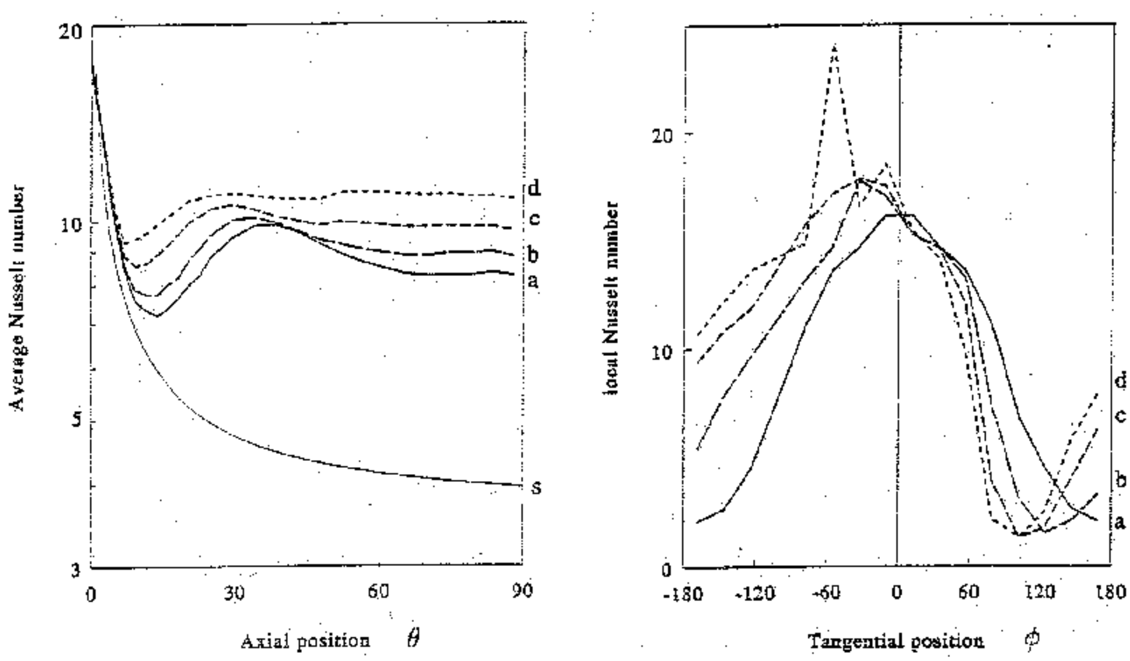

Figure 3: a: $\sqrt{Gr}/De = 0$, b: $\sqrt{Gr}/De = 2.37$, c: $\sqrt{Gr}/De = 3.33$, d: $\sqrt{Gr}/De = 4.09$, s: horizontal straight pipe. Left: Nusselt number, averaged over the pipe circumference, as a function of position along the bend. Right: variation of the the Nusselt number over the pipe circumference, at position $\theta = 29°$ along the bend. $\phi = 0$ corresponds the outer side of the bend, and $-180 < \phi < 0$ the bottom part. Images taken from Sillekens *et al.* (1994).

References

[1] S. A. Berger, L. Talbot, and L.-S Yao. Flow in curved pipes. *Ann. Rev. Fluid Mech.*, 15:461–512, 1983.

[2] B. R. Morton. Laminar convection in uniformly heated horizontal pipes at low rayleigh numbers. *Quart. Journ. Mech. and Applied Math.*, 12(4), 1959.

[3] P. Naphon and S. Wongwises. A review of flow and heat transfer characteristics in curved tubes. *Renewable and Sustainable Energy Reviews*, 10:463–490, 2004.

[4] J. Prusa and L.-S. Yao. Numerical solution for fully developed flow in heated curved tubes. *J. Fluid Mech.*, 123:503–522, 1982.

[5] J. J. M. Sillekens, C. C. M. Rindt, and A. A. van Steenhoven. Mixed convection in a 90 degrees horizontal bend. *Proceedings of the 10th International Heat Transfer Conference, Brighton UK*, pages 567–572, 1994.

[6] L.-S. Yao and S. A. Berger. Flow in heated curved pipes. *J. Fluid Mech.*, 88(2):339–354, 1978.