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A Study On The Heat Transfer of Nanofluids in Pipes

Koh Kai Liang Peter
Dept. of Energy Sciences, Faculty of Engineering,
Lund University, Box 118, 22100 Lund, Sweden

Abstract

The remarkable thermophysical properties that nanofluids possess give it rising potential as a working fluid in many industries. This paper aims to investigate whether the use of nanofluids as a working fluid, as opposed to using water/oil, will reduce the pipe dimensions in an industrial set up. An understanding of the thermal conductivity in nanofluids is discussed before a suitable heat analysis method is developed to give relationships for the Nusselt number. After which, an analysis on the thin-walled pipe with constant wall temperature is applied to evaluate whether the required pipe dimensions are smaller if nanofluids are employed as the working fluid.

Nomenclature

c_p	Specific Heat Capacity
D	Diameter
D_x	Thermal dispersion coefficient
h	Heat transfer coefficient
k	Thermal Conductivity
L	Length
$LMTD$	Log mean temperature difference
$\dot{m}$	Mass Flow Rate
Nu	Nusselt number
Pe	Pecelt Number
R	Radius
r	Radial variable
T	Temperature
t	Time
u	Axial velocity
x	Axial variable

Greek Symbols

φ	Volume Fraction
λ	Thickness to Radius ratio
α	Thermal Diffusivity
ρ	Density

Subscripts

b	Bulk
bc	Brownian Motion
eff	Effective
f	Fluid
in	Inlet
$nano$	Nanoparticle
out	Outlet
p	Particle
s	Stationary
w	Wall

1. Introduction

Nanofluids comprise of suspensions of nanoparticles with at least one of their principal dimensions smaller than 100nm [2]. Research has shown that nanofluids, in comparison to base fluids like water/oil, possess enhanced thermophysical properties such as thermal conductivity and convective heat transfer coefficient. The reasons why nanofluids possess enhanced thermal properties are as follows [1]:

1) The suspended nanoparticles increase the relative surface area of the fluid.
2) The suspended nanoparticles increasing the effective thermal conductivity of the fluid.
3) The interaction and collision among particles, fluid and the flow passage surface are intensified.
4) The mixing fluctuation and turbulence of the fluid are intensified.
5) The dispersion of nanoparticles flattens the transverse temperature gradient of the fluid.

There are also various factors such as particle shape & size, clustering of particles, temperature of fluid and dissociation of surfactant that affect the thermal properties of nanofluids. This paper will highlight some of them.

2. Problem Statement

There are numerous industrial applications for pipes to heat up fluids and quite commonly, water is used. Based on the working fluid to be used, it is often desirable for companies to optimize the size of the heat pipe for their relevant applications. Optimizing the pipe diameter D and length L is of paramount importance because it can potentially reduce their basic costs and maintenance costs. Nanofluids are potentially a suitable alternative to water as a working fluid.

This claim was similarly highlighted in [1] and thus the aim of this paper is to find out whether nanofluids are truly capable of reducing pipe dimensions in an industrial system. This paper will only focus on thin-walled pipes with a constant wall temperature.

3. Literature Survey

Many models for predicting the thermal conductivity has been developed over the last 50 years. It began with the Hamilton & Crosser Model (1962) and was slowly modified over time. Most recently, the modified model from Azari et al. (2014) [3] appears to have a close connection to the experimental data.

A model to analyse the heat transfer of nanofluids was also developed by Taylor (1953). He introduced a dispersed model of the energy equation to account for the random movement of particles in the main flow. Y. Xuan et al. (2000) [1] utilizes a modified version of this model for his heat transfer analysis.

Although there are lot of studies on the thermal conductivity and heat transfer analysis, there are very few studies on the specific heat capacity on nanofluids. One model tested by Hanley et al. (2012) [5] seems to have a reasonable correlation with experimental data.

4. Model for Thermal Conductivity of Nanofluids

Nanofluids are two-component mixtures comprising of a base fluid and nanoparticles. The thermal conductivity of the nanofluid will thus have a relationship with their individual thermal conductivities. One of the first models developed to evaluate the thermal conductivity of two-component mixtures is the Hamilton & Crosser Model (1962). It is expressed by

$$\frac{k_s}{k_f} = \frac{k_p + 2k_f - 2\varphi(k_f - k_p)}{k_p + 2k_f + \varphi(k_f - k_p)} \tag{1}$$

where k_f & k_p are the thermal conductivities of the fluid and particle respectively, and φ is the volume fraction of the nanoparticles in the nanofluid.

This model has been highly successful at predicting the thermal conductivity of fluids with stationary nanoparticles, however, over the years many developments have been made to further improve on this model. Azari et al. (2014) proposed that the model should include effects due to the nanolayer thickness, effects due to Brownian motion and the size distribution of the nanoparticles [3]. In their research paper, they published that their model agrees very well with the experimental data. This paper will therefore utilize the model that Azari et al. (2014) has developed.

The first modification to the Hamilton-Crosser model is to take into consideration the nanolayer thickness. The interface between solid and liquid can be viewed as a thin nanolayer, thus it will have semi-solid material properties. The volume fraction in the above formula will need to be adjusted to factor in the thin nanolayer. The modified equation is given by

$$\frac{k_s}{k_f} = \frac{k_p + 2k_f - 2\varphi(1+\lambda_{nano})^3(k_f - k_p)}{k_p + 2k_f + \varphi(1+\lambda_{nano})^3(k_f - k_p)} \tag{2}$$

where λ_{nano} is the ratio between the thickness of the nanolayer, t_p, and the radius of the nanoparticle r_p.

The two other important factors to account for are Brownian motion and the size distribution of the nanoparticles. Utilizing previous research work from the scientific community, Azari et al. (2014) derived an equation that included these 2 effects. The equation he derived was

$$k_{bc} = R\frac{M_1}{M_2}M\left(\frac{2}{\gamma_p} + \frac{Z}{\gamma_p^2}\right) \tag{3}$$

$$R = 2.1d_f \tag{4}$$

$$D_f = d - \frac{\ln\varphi}{\ln\left(\frac{\gamma_{min}}{\gamma_{max}}\right)} \tag{5}$$

$$M_1 = \left[\left(\frac{\gamma_{max}}{\gamma_{min}}\right)^{1-D_f} - 1\right]^2 \tag{6}$$

$$M_2 = \left(\frac{\gamma_{max}}{\gamma_{min}}\right)^{2-D_f} - 1 \tag{7}$$

$$M = \frac{D_f(2-D_f)}{\left(1-D_f\right)^2} \tag{8}$$

$$Z = \frac{k_B T\rho C_p}{\pi\mu_{eff}k_f} \tag{9}$$

where d_f is the diameter of the base fluid molecule, D_f is the fractal dimensions for particles, γ_p , γ_{min} , γ_{max} are the average, minimum & maximum diameters of the nanoparticles respectively, k_B is the Boltzmann constant, T is the temperature, ρ is the density of the nanoparticle, μ_{eff} is the dynamic viscosity of the base fluid and C_p is the specific heat capacity.

According to a group of researchers that was mentioned in [3], the total thermal conductivity of the nanofluid is the sum of the thermal conductivity of the stationary particles and the thermal conductivity due to Brownian motion. It can be expressed as

$$k_{eff} = k_s + k_{bc} \tag{10}$$

The model Azari et al. (2014) developed was then tested against the experiments for copper (II) oxide in water and other existing models predicting thermal conductivity. Quite commonly, the transient hot wire method is used for measuring the thermal conductivity. A detailed description of how it is done is given in [4]. In the experiment, two different average particle diameters of copper (II) oxide were used, one with diameter 18nm and the other with diameter 23.6nm. It is observed that the values the model predict comes very close to experimental values as indicated by the arrow in Fig. 1. & Fig. 2.

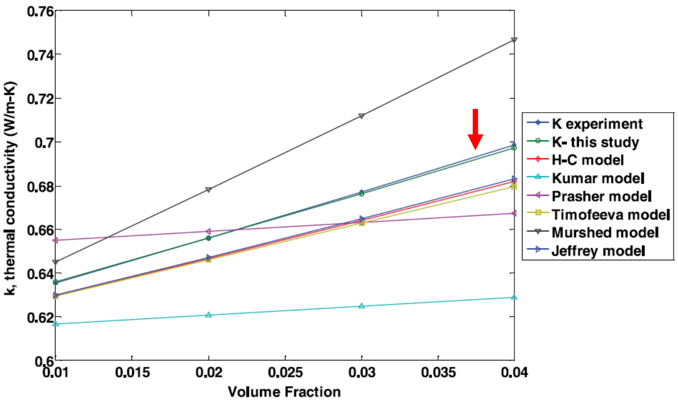

Fig. 1. Comparison of effective thermal conductivity values for CuO (18nm)/water

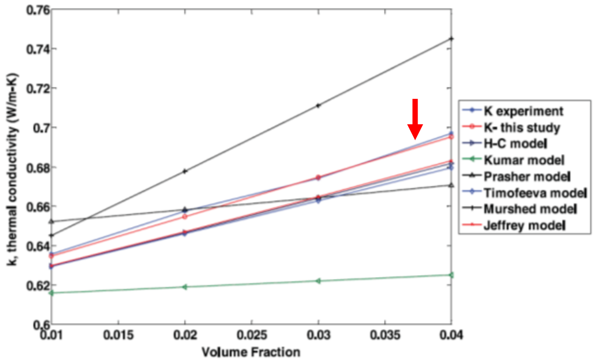

Fig. 2. Comparison of effective thermal conductivity values for CuO (23.6nm)/water

5. Factors Affecting Thermal Conductivity

5.1 Volume Fraction

From Fig. 1. & Fig. 2. we observe that increasing the volume fraction of nanoparticles, the thermal conductivity increases. This increase is due to the fact that the nanoparticles give the nanofluid extra surface area for heat transfer.

5.2 Nanolayer Thickness

The thermal conductivity of the nanolayer is a function of k_p & k_f since the nanolayer has semi-solid properties. As the thickness of the nanolayer increases, the effective volume fraction increases, thus increasing the effective thermal conductivity of the nanofluid.

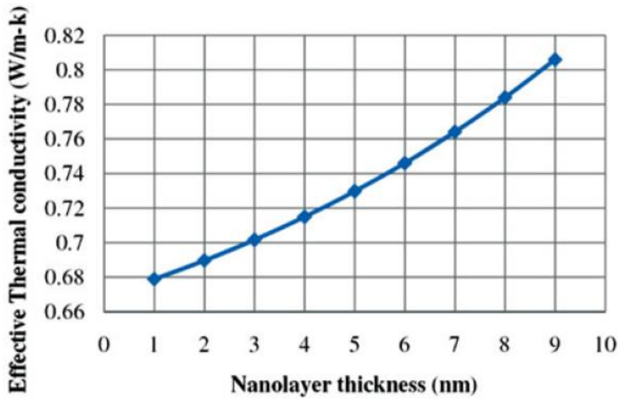

Fig. 3. Effective thermal conductivity versus nanolayer thickness for CuO (18nm)/water

5.3 Particle Diameter

The thermal conductivity of nanofluids decreases as the particle diameter is increased. This is because as the particle size increases, the ratio of its surface area to its volume decreases [1].

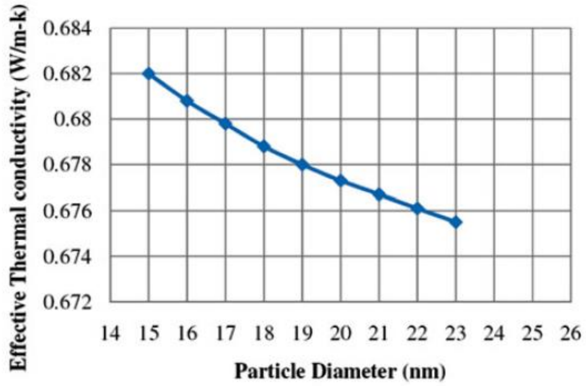

Fig. 4. Effective thermal conductivity versus particle diameter for CuO (18nm)/water at constant volume fraction & temperature.

5.4 Temperature

Temperature plays an important role because at high temperatures, particles are at higher energy levels and they move very fast to interact, thereby increasing the thermal conductivity of nanofluids.

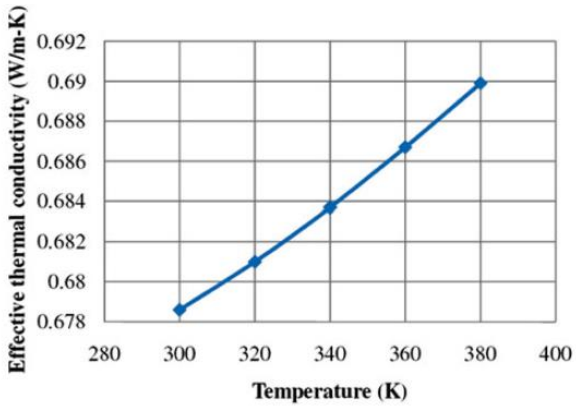

Fig. 5. Effective thermal conductivity versus temperature for CuO (18nm)/water

6. Heat Transfer Analysis

The model to be used for heat transfer analysis is the dispersion model of the energy equation as described in [1]. The reason it is chosen is because it takes into account the random movement of particles. This random movement, as discussed earlier in terms of Brownian motion, produces convection-like effects at the nanoscale [3]. In addition, the chaotic movement of the particles strengthens the energy transport process [1].

The governing partial differential equation of a nanofluid flowing inside a tube is given by

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \left(\alpha_{\text{eff}} + \frac{D_x}{(\rho c_p)_{\text{eff}}}\right)\frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left[\left(\alpha_{\text{eff}} + \frac{D_r}{(\rho c_p)_{\text{eff}}}\right) r\frac{\partial T}{\partial r}\right] \quad (11)$$

where D_x is the thermal dispersion coefficient to account for hydrodynamic dispersion and irregular movement of ultra fine particles.

If the axial temperature gradient is neglected, the above P.D.E. becomes

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left[\left(\alpha_{\text{eff}}+\frac{D_r}{(\rho c_p)_{\text{eff}}}\right)r\frac{\partial T}{\partial r}\right] \quad (12)$$

The boundary conditions for the flow are

$$T|_{r=R}=T_w \quad (13)$$

$$T|_{x=0}=T_0 \quad (14)$$

The steady state solution to the P.D.E is

$$\frac{T-T_w}{T_0-T_w}=2\sum_{m=1}^{\infty}e^{-\beta_m^2/\overline{Pe}}\frac{J_0(\beta_m\bar{r})}{J_1(\beta_m)\beta_m} \quad (15)$$

Where, $\bar{r}=r/R$, $\bar{x}=x/L$, $Nu=2Rh/k_{eff}^*$, $Pe^*=uL/\alpha_{eff}^*$, $\overline{Pe}=Pe^*(R/L)^2$ and eigenvalues β_m's are the positive roots of the following equation:

$$J_0(\beta_m)=0 \quad (16)$$

According to the heat transfer relation

$$-k_{\text{eff}}^*\frac{\partial T}{\partial r}\bigg|_{r=R}=h(T_b-T_w) \quad (17)$$

an analytical relation for the Nusselt Number can be derived (Fig. 7.)

$$Nu=\frac{\sum_{m=1}^{\infty}e^{-\beta_m^2/\overline{Pe}}}{\sum_{m=1}^{\infty}e^{-\beta_m^2/\overline{Pe}}/\beta_m^2} \quad (18)$$

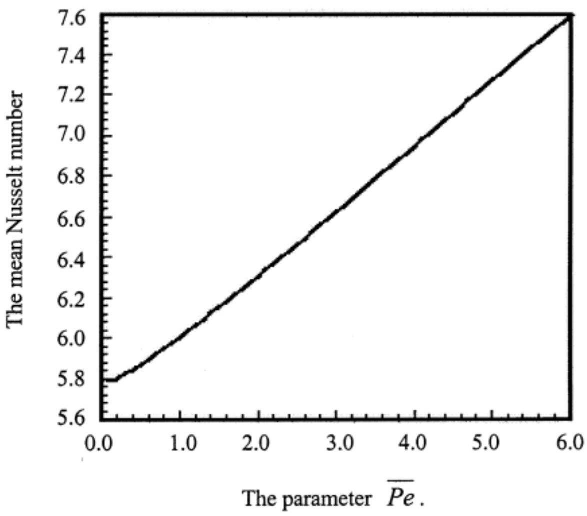

Fig. 7. The mean Nusselt number v.s. the parameter $\overline{Pe}$.

A relation for the specific heat capacity [5] of the nanofluid is given by

$$c_{p,eff}=\frac{\varphi(\rho c_p)_p+(1-\varphi)(\rho c_p)_f}{\varphi\rho_p+(1-\varphi)\rho_f} \quad (19)$$

Although thermal conductivity increases with increasing volume fraction of nanoparticles, the specific heat capacity does not follow the same pattern. The specific heat capacity will instead decrease with increasing volume fraction. Several experiments conducted by Hanley et al. (2012) prove this relationship [5]. Fig. 8. shows one of the graphs from the experiment. It can be seen that the specific heat capacity decreases when volume fraction increases.

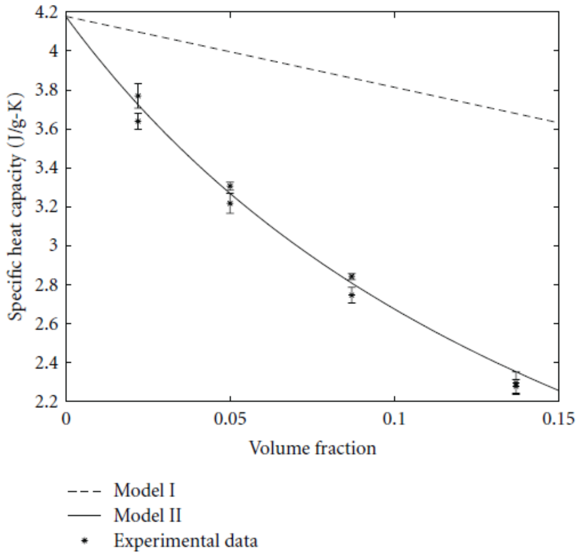

Fig. 8. Specific Heat Capacity of Copper-Oxide/Water with Different Volume Fractions at 35°C

7. Analysis of pipe flow with constant wall temperature

Supposing in a pipe, the wall temperature is T_w, the inlet & outlet temperatures are T_{in} & T_{out} respectively, and the mass flow rate is $\dot{m}$. The goal is to find out whether using nanofluids as a working fluid will reduce pipe dimensions when all the aforementioned parameters are kept constant. A benchmark comparison can be made against water. There are 2 cases for comparison: (a) Find D for a constant L, (b) Find L for a constant D.

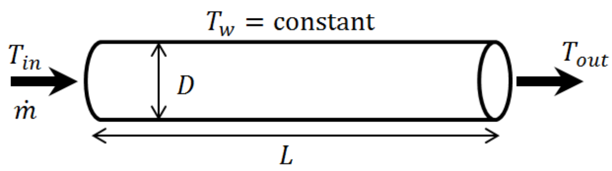

Fig. 9. Schematic Diagram of a Pipe Flow Problem

For constant wall temperature,

$$\dot{m}c_{p,eff}(T_{out}-T_{in})=h(\pi DL)LMTD \quad (20)$$

$$LMTD=\frac{(T_w-T_{in})-(T_w-T_{out})}{\ln\frac{T_w-T_{in}}{T_w-T_{out}}} \quad (21)$$

For water, the relations for Nusselt Number are given by

Laminar $\quad \overline{Nu}_D=\frac{hD}{k}=3.66 \quad (22)$

Turbulent $\quad \overline{Nu}_D=\frac{hD}{k}=0.023\,Re_D^{0.8}Pr^{0.4} \quad (23)$

8. Predicted Results

In relation to equation (20), the 2 variables that are affected by the use of nanofluids are the heat transfer coefficient & specific heat capacity. This will in turn cause the pipe dimensions of D & L to vary depending on the chosen working fluid.

Based on the studies conducted by Hanley et al. (2012), the specific heat capacity of a water-based nanofluid is going be lower than water [5]. This trend is observed in Fig. 8.

The heat transfer coefficient depends on both the diameter and the thermal conductivity of the fluid as shown in equations (22) & (23). The thermal conductivity in a nanofluid is higher than in water. This means that for a given diameter, there is a high chance that the heat transfer coefficient in a nanofluid will be higher than water.

In a laminar flow regime, we observe that the Nusselt number in a nanofluid will always be larger than that of water. With reference from Fig. 7., he Nusselt number for a nanofluid begins at a value of 5.8 and increases with increasing velocity. The Nusselt number for water, on the other hand, will stay constant at 3.66. Overall, what this means is that the heat transfer coefficient for a nanofluid will definitely be larger than that of water. Hence, the required dimensions for D & L will be always be smaller in the case of a nanofluid in the laminar flow regime.

In a turbulent flow regime, we realize that the comparison between the Nusselt numbers for water and nanofluids is not as simple. This is because the Nusselt number for the nanofluid is a function of the mean Pecelt Number as seen in Fig. 7. And the mean Pecelt Number itself is a function of other variables as in equation (15). This can be summarized below

Nanofluid: $$Nu = \frac{hD}{k_{eff}} = f(\overline{Pe}) = f\left(\frac{uL}{\alpha^*_{eff}}\left(\frac{D}{2L}\right)^2\right) \tag{24}$$

Water: $$\overline{Nu}_D = 0.023\, Re_D^{0.8} Pr^{0.4} = f(u^{0.8} D^{0.8}) \tag{25}$$

From a mathematical point of view, it would be sensible to conclude that the Nusselt number for a nanofluid will always be greater than that for water. This is because for a nanofluid, the Nusselt number is proportional to the velocity and the square of the diameter. Whereas in water, both velocity and diameters are proportional to the power of 0.8. This means that the when the mass flow rate is increased, the Nusselt number for a nanofluid will increase at a faster rate than water.

Thus, one can conclude that in all cases, laminar or turbulent, the heat transfer coefficient will be greater in nanofluids than in water. This means that the required pipe dimensions for using nanofluids as a working fluid will always be smaller than using water as a working fluid. This paper is unable to conclude how much smaller the dimensions will be because of a lack of thermophysical data on nanofluids.

9. Key Issues

Although the above concluded that using nanofluids will help to cut down on pipe size, it is necessary to understand that selected models and assumptions were used. The research of nanofluids is still plagued with many issues. The first major issue is that a lot of experimental work has been contradictory. Some studies cite enhanced heat transfer performance, while others show no or less enhancement. The reason this happens is because there is a lack of understanding of the complex nature of nanofluids [6]. One such example is a research done by Z. Wu et al. (2013). In their paper, they studied the heat transfer of nanofluids in double pipe helical heat exchangers and concluded that using nanofluids had an insignificant effect [7]. The other major issues are mostly practical in nature [6] and are discussed in the following section.

9.1 Production Challenges

Producing nanofluids result in many technical and financial difficulties. One of the technical challenges involved is that nanoparticles tend to undergo reduction reactions and ion exchange with the base fluid compound. The effective amount of nanoparticles inside the base fluid will therefore be different from the original amount put in. This then will greatly affect data on thermal conductivity. Nanofluids are also extremely costly to produce and hence in most experiments, only small quantities are used. Large scale testing would be required in order to fully assess their relevance for industrial applications.

9.2 High Viscosity

Many experiments have reported that the viscosity of water-based nanofluids tend to increase with increasing volume fraction. This means that beyond a certain volume fraction, it becomes impractical to use a nanofluid as a working fluid. The reason is because with a higher viscosity, more pumping power is required, which translates to more costs. The pressure drop is also very high for fluids with high viscosity.

9.3 Lower Specific Heat

As mentioned in section 6, the specific heat capacity of nanofluids tend to decrease with increasing volume fraction. This makes nanofluids unsuitable for cooling applications where a high specific heat capacity is required to remove more heat.

10. Conclusion

An understanding on the thermal conductivity of nanofluids was first established. The thermal conductivity of nanofluids is observed to increase with increase volume fraction predominantly because it provides an increase in relative surface area for heat transfer. There are also other factors that affect the thermal conductivity such as nanolayer thickness, particle diameter and temperature. A heat transfer analysis for nanofluids undergoing pipe flow was then developed. In the formulation, a dispersion model of the energy equation was utilized to account for the random motion of particles in a flow. The analytical results for the Nusselt number of nanofluids was then obtained. This Nusselt number for nanofluids was compared against the Nusselt number for water. It was concluded that in all flow regimes, the Nusselt number will always be greater and hence the dimensions of a pipe using nanofluids as a working fluid will be smaller as compared to using water as a working fluid. Due to the lack of thermophysical data for nanofluids, it was not possible to quantify how much smaller the pipe dimensions will be.

Although nanofluids have great potential to replace water, there are still many hurdles for it to cross. For one, the cost of producing a nanofluid is very high. Nanofluid viscosity also tends to increase with increasing volume fraction. So beyond a certain point, any increase in thermal conductivity will be redundant. Lastly, the specific heat capacity decreases with increasing volume fraction, this makes nanofluids less suited for cooling applications.

11. References

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