Chapter 4 Incompressible Flow over Airfoils

The vortex panel method is an important numerical technique for the solution of the inviscid, incompressible flow over bodies of arbitrary shape, thickness, and angle of attack. For panels of constant strength, the governing equations are

\[ V_\infty \cos \beta_i - \sum_{j=1}^{n} \frac{y_j}{2\pi} \int \frac{\partial \theta_{ij}}{\partial n_i} \, ds_j = 0 \quad (i = 1, 2, \ldots, n) \]

and

\[ \gamma_i = -\gamma_{i-1} \]

which is one way of expressing the Kutta condition for the panels immediately above and below the trailing edge.

4.17 PROBLEMS

4.1 Consider the data for the NACA 2412 airfoil given in Figure 4.10. Calculate the lift and moment about the quarter chord (per unit span) for this airfoil when the angle of attack is 4° and the freestream is at standard sea level conditions with a velocity of 15 m/s. The chord of the airfoil is 0.6 m.

4.2 Consider an NACA 2412 airfoil with a 2-m chord in an airstream with a velocity of 50 m/s at standard sea level conditions. If the lift per unit span is 1535 N, what is the angle of attack?

4.3 Starting with the definition of circulation, derive Kelvin’s circulation theorem, Equation (4.11).

4.4 Starting with Equation (4.35), derive Equation (4.36).

4.5 Consider a thin, symmetric airfoil at 1.5° angle of attack. From the results of thin airfoil theory, calculate the lift coefficient and the moment coefficient about the leading edge.

4.6 The NACA 4412 airfoil has a mean camber line given by

\[ \frac{z}{c} = \begin{cases} 
0.25 \left[ 0.8 \frac{x}{c} - \left( \frac{x}{c} \right)^2 \right] & \text{for } 0 \leq \frac{x}{c} \leq 0.4 \\
0.111 \left[ 0.2 + 0.8 \frac{x}{c} - \left( \frac{x}{c} \right)^2 \right] & \text{for } 0.4 \leq \frac{x}{c} \leq 1 
\end{cases} \]

Using thin airfoil theory, calculate

(a) \( \alpha_{L=0} \) (b) \( c_l \) when \( \alpha = 3° \)

4.7 For the airfoil given in Problem 4.6, calculate \( c_{m, x=0} \) and \( x_{cp}/c \) when \( \alpha = 3° \).

4.8 Compare the results of Problems 4.6 and 4.7 with experimental data for the NACA 4412 airfoil, and note the percentage difference between theory and experiment. (Hint: A good source of experimental airfoil data is Reference 11.)
4.9 Starting with Equations (4.35) and (4.43), derive Equation (4.62).

4.10 For the NACA 2412 airfoil, the lift coefficient and moment coefficient about the quarter-chord at a $-6^\circ$ angle of attack are $-0.39$ and $-0.045$, respectively. At $4^\circ$ angle of attack, these coefficients are $0.65$ and $-0.037$, respectively. Calculate the location of the aerodynamic center.

4.11 Consider again the NACA 2412 airfoil discussed in Problem 4.10. The airfoil is flying at a velocity of $60$ m/s at a standard altitude of $3$ km (see Appendix D). The chord length of the airfoil is $2$ m. Calculate the lift per unit span when the angle of attack is $4^\circ$.

4.12 For the airfoil in Problem 4.11, calculate the value of the circulation around the airfoil.

4.13 In Section 3.15 we studied the case of the lifting flow over a circular cylinder. In real life, a rotating cylinder in a flow will produce lift; such real flow fields are shown in the photographs in Figures 3.34(b) and (c). Here, the viscous shear stress acting between the flow and the surface of the cylinder drags the flow around in the direction of rotation of the cylinder. For a cylinder of radius $R$ rotating with an angular velocity $\omega$ in an otherwise stationary fluid, the viscous flow solution for the velocity field obtained from the Navier-Stokes equations (Chapter 15) is

$$V_\theta = \frac{R^2 \omega}{r}$$

where $V_\theta$ is the tangential velocity along the circular streamlines and $r$ is the radial distance from the center of the cylinder. (See Schlichting, Boundary-Layer Theory, 6th ed., McGraw-Hill, 1968, page 81.) Note that $V_\theta$ varies inversely with $r$ and is of the same form as the inviscid flow velocity for a point vortex given by Equation (3.105). If the rotating cylinder has a radius of $1$ m and is flying at the same velocity and altitude as the airfoil in Problem 4.11, what must its angular velocity be to produce the same lift as the airfoil in Problem 4.11? (Note: You can check your results with the experimental data for lift on rotating cylinders in Hoerner. Fluid-Dynamic Lift, published by the author, 1975, p. 21–44, Fig. 5.)

4.14 The question is often asked: Can an airfoil fly upside-down? To answer this, make the following calculation. Consider a positively cambered airfoil with a zero-lift angle of $-3^\circ$. The lift slope is $0.1$ per degree. (a) Calculate the lift coefficient at an angle of attack of $5^\circ$. (b) Now imagine the same airfoil turned upside-down, but at the same $5^\circ$ angle of attack as part (a). Calculate its lift coefficient. (c) At what angle of attack must the upside-down airfoil be set to generate the same lift as that when it is right-side-up at a $5^\circ$ angle of attack?

4.15 The airfoil section of the wing of the British Spitfire of World War II fame (see Figure 5.19) is a NACA 2213 at the wing root, tapering to a NACA 2205 at the wing tip. The root chord is $2.5$ m. The measured profile drag coefficient of the NACA 2213 airfoil is $0.006$ at a Reynolds number of
9 \times 10^6. Consider the Spitfire cruising at an altitude of 5,400 m. (a) At what velocity is it flying for the root chord Reynolds number to be 9 \times 10^6? (b) At this velocity and altitude, assuming completely turbulent flow, estimate the skin-friction drag coefficient for the NACA 2213 airfoil, and compare this with the total profile drag coefficient. Calculate the percentage of the profile drag coefficient that is due to pressure drag. 

Note: Assume that \mu varies as the square root of temperature, as first discussed in Section 1.8.

4.16 For the conditions given in Problem 4.15, a more reasonable calculation of the skin friction coefficient would be to assume an initially laminar boundary layer starting at the leading edge, and then transitioning to a turbulent boundary layer at some point downstream. Calculate the skin-friction coefficient for the Spitfire’s airfoil described in Problem 4.15, but this time assuming a critical Reynolds number of 10^6 for transition.
Results from classical lifting-line theory:

**Elliptic wing:**

Downwash is constant:

\[ w = \frac{\Gamma_0}{2b} \]  
(5.35)

\[ \alpha_i = \frac{C_L}{\pi AR} \]  
(5.42)

\[ C_{D,i} = \frac{C_L^2}{\pi AR} \]  
(5.43)

\[ \alpha = \frac{a_0}{1 + a_0/\pi AR} \]  
(5.69)

**General wing:**

\[ C_{D,i} = \frac{C_L^2}{\pi AR}(1 + \delta) = \frac{C_L^2}{\pi e AR} \]  
(5.61) and (5.62)

\[ \alpha = \frac{a_0}{1 + (a_0/\pi AR)(1 + \tau)} \]  
(5.70)

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For low-aspect-ratio wings, swept wings, and delta wings, lifting-surface theory must be used. In modern aerodynamics, such lifting-surface theory is implemented by the vortex panel or the vortex lattice techniques.

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### 5.10 PROBLEMS

5.1 Consider a vortex filament of strength $\Gamma$ in the shape of a closed circular loop of radius $R$. Obtain an expression for the velocity induced at the center of the loop in terms of $\Gamma$ and $R$.

5.2 Consider the same vortex filament as in Problem 5.1. Consider also a straight line through the center of the loop, perpendicular to the plane of the loop. Let $A$ be the distance along this line, measured from the plane of the loop. Obtain an expression for the velocity at distance $A$ on the line, as induced by the vortex filament.

5.3 The measured lift slope for the NACA 23012 airfoil is $0.1080$ degree$^{-1}$, and $\alpha_{L=0} = -1.3^\circ$. Consider a finite wing using this airfoil, with $AR = 8$ and taper ratio $= 0.8$. Assume that $\delta = \tau$. Calculate the lift and induced drag coefficients for this wing at a geometric angle of attack $= 7^\circ$. 
5.4 The Piper Cherokee (a light, single-engine general aviation aircraft) has a wing area of 15.8 m² and a wing span of 9.6 m. Its maximum gross weight is 11 kN. The wing uses an NACA 65-415 airfoil, which has a lift slope of 0.1033 degree⁻¹ and \( \alpha_s = -3^\circ \). Assume \( \tau = 0.12 \). If the airplane is cruising at 190 km/h at standard sea level at its maximum gross weight and is in straight-and-level flight, calculate the geometric angle of attack of the wing.

5.5 Consider the airplane and flight conditions given in Problem 5.4. The span efficiency factor \( e \) for the complete airplane is generally much less than that for the finite wing alone. Assume \( e = 0.64 \). Calculate the induced drag for the airplane in Problem 5.4.

5.6 Consider a finite wing with an aspect ratio of 6. Assume an elliptical lift distribution. The lift slope for the airfoil section is 0.1/degree. Calculate and compare the lift slopes for (a) a straight wing, and (b) a swept wing, with a half-chord line sweep of 45 degrees.

5.7 Repeat Problem 5.6, except for a lower aspect ratio of 3. From a comparison of the results from these two problems, draw some conclusions about the effect of wing sweep on the lift slope, and how the magnitude of this effect is affected by aspect ratio.

5.8 In Problem 1.19 we noted that the Wright brothers, in the design of their 1900 and 1901 gliders, used aerodynamic data from the Lilienthal table given in Figure 1.65. They chose a design angle of attack of 3 degrees, corresponding to a design lift coefficient of 0.546. When they tested their gliders at Kill Devil Hills near Kitty Hawk, North Carolina, in 1900 and 1901, however, they measured only one-third the amount of lift they had originally calculated on the basis of the Lilienthal table. This led the Wrights to question the validity of Lilienthal’s data, and this cast a pall on the Lilienthal table that has persisted to the present time. However, in Reference 62 this author shows that the Lilienthal data are reasonably valid, and that the Wrights misinterpreted the data in the Lilienthal table in three respects (see pages 209–216 of Reference 62). One of these respects was the difference in aspect ratio. The Wrights’ 1900 glider had rectangular wings with an aspect ratio of 3.5, whereas the data in the Lilienthal table were taken with a wing with an ogival planform tapering to a point at the tip and with an aspect ratio of 6.48. The Wrights seemed not to appreciate the aerodynamic importance of aspect ratio at the time, and even if they had, there was no existing theory that would have allowed them to correct the Lilienthal data for their design. (Prandtl’s lifting line theory appeared 18 years later.) Given just the difference in aspect ratio between the Wrights’ glider and the test model used by Lilienthal, what value of lift coefficient should the Wrights have used instead of the value of 0.546 they took straight from the table? (Note: There are two other misinterpretations by the Wrights that resulted in their calculation of lift being too high; see Reference 62 for details.)
5.9 Consider the Supermarine Spitfire shown in Figure 5.19. The first version of the Spitfire was the Mk I, which first flew in 1936. Its maximum velocity is 580 km/h at an altitude of 5.6 km. Its weight is 26.4 kN, wing area is 22.5 m², and wing span is 10.8 m. It is powered by a supercharged Merlin engine, which produced 790 kW at 5.6 km. (a) Calculate the induced drag coefficient of the Spitfire at the flight condition of \( V_{\text{max}} \) at 5.6 km. (b) What percentage of the total drag coefficient is the induced drag coefficient? Note: To calculate the total drag, we note that in steady level flight of the airplane, \( T = D \), where \( T \) is the thrust from the propeller. In turn, the thrust is related to the power by the basic mechanical relation \( TV_\infty = P \), where \( P \) is the power supplied by the propeller-engine combination. Because of aerodynamic losses experienced by the propeller, \( P \) is less than the shaft power provided by the engine by a ratio, \( \eta \), defined as the propeller efficiency. See Chapter 6 of Reference 2 for more details. For this problem, assume the propeller efficiency for the Spitfire is 0.9.

5.10 If the elliptical wing of the Spitfire in Problem 5.9 were replaced by a tapered wing with a taper ratio of 0.4, everything else remaining the same, calculate the induced drag coefficient. Compare this value with that obtained in Problem 5.9. What can you conclude about the relative effect of planform shape change on the drag of the airplane at high speeds?

5.11 Consider the Spitfire in Problem 5.9 on its landing approach at sea level with a landing velocity of 112 km/h. Calculate the induced drag coefficient for this low-speed case. Compare your result with the high-speed case in Problem 5.9. From this, what can you conclude about the relative importance of the induced drag coefficient at low speeds compared to that at high speeds?