

# HEAT TRANSFER IN LIQUID FILM OVER THE SURFACE OF A ROTATING DISK SUBJECTED TO IMPINGING JET

Ali Alhelfi  
 Dept. of Energy Sciences, Faculty of Engineering,  
 Lund University, Box 118, 22100 Lund, Sweden

## ABSTRACT

This work is theoretical analyses of heat transfer across film a flowing over the surface of a rotating disk. This field has wide industrial applications in Turbomachinery, chemical, and food industries.

An analytical study is developed to investigate the heat transfer characteristics in the liquid film due to jet impinging at the center of the disk. A theoretical model is based on heat balance and energy equations developed for a rotating disk heated by a constant heat flux and the free surface are subjected to the surrounding.

Local Nusselt number and temperature were found for different selected parameters such as Prandtl number and Reynolds number. Nusselt number obtained from the present model was compared with that obtained by Thomas et al., and the agreement between the results was acceptable.

## NOMENCLATURE

A	Area	$m^2$
Bi	Bi ot number	
$C_p$	Specific heat at constant pressure	$kJ/kg.K$
h	Heat transfer coefficient	$W/m^2.K$
k	Thermal conductivity	$W/m.K$
$\dot{m}$	Mass flow rate	$kg/s$
Nu	Nusselt number	$h\delta/k$
Pr	Prandtl number	
$q''$	Heat flux	$W/m^2$
Q	Volumetric flow rate	$m^3/s$
r	Radial coordinate	m
Re	Reynolds number	

## Greek Symbols

$\Delta$	Difference	
$\delta$	Film thickness	m
$\alpha$	Thermal diffusivity	$m^2/s$
$\rho$	Density	$kg/m^3$
$\omega$	Angular velocity	rad/s
$\mu$	Dynamic viscosity	$N.s/m^2$
$\nu$	Kinematic viscosity	$m^2/s$
$\zeta$	Dimensionless coordinate parallel to the disk,	$r/\delta$

## Superscripts

b	Bulk
i	At the inlet
c	Cross section
o	Out of the impingement jet
w	Surface of the disk
$\infty$	Ambient

## SUPERSCRIPTS

-	Average
+	Dimensionless

## INTRODUCTION

There are various kinds of rotating mechanical devices, which can be met in industrial application. Turbomachinery, rotating-disk contactors, and rotating heat exchangers are just a few examples [1]. So, various filed of engineering contain rotating disk systems, these system include single disks in unbounded media (free disks), rotor-stator systems, and sets of disks rotating in opposite direction or in the same direction. In such systems, the rotational motion of the medium is often accompanied by forced axial or radial flows. The latter flows can be either centrifugal or centripetal.

The interest in this subject has increased considerably over the past few years. Major gas-turbine manufacturers in Europe, U.S.A., and Japan currently funding a large numbers of research projects in that direction. In the case of gas turbine disk cooling, the coolant is usually air supplied from the compressor section of the engine, and its use generally penalizes the engine cycle performance. Thus the objective of disk cooling scheme design is to provide the minimum necessary coolant consistent with desired engine life and reliability. The air supplied to turbine disk cooling jets is usually viewed as having two functions. First, it is expected that the high heat transfer rates normally associated with impingement on to stationary surfaces will be present even when the surface is rotating, and that they will probably be enhanced by the rotation and correspondingly increased relative velocity between jet and surface. Second, it is anticipated that if multiple jets are used and properly placed, they can prevent the undesired radial inflow of hot combustion gases into the cavity between the disk and adjacent stationary shroud [2]. Also, an investigation, supported by the National Aeronautics and Space Administration (NASA-U.S.A), was initiated with the primary objective of determining heat transfer behavior for a variety of uniformly spaced impingement array configuration which model those of interest in current and contemplated gas turbine air foil cooling applications [3].

Furthermore, in many gas-liquid heat exchangers, heat transfer performance is controlled by thermal resistance on the gas side. Two remedies are available to reduce this resistance: by increasing heat transfer area through implementation of fins, and by enhancing the heat transfer coefficient through increasing the flow or generating secondary flow. Various means have been applied to produce secondary flows, for example oscillation of the bulk flow, vibration or rotation of the surface, introduction of acoustic waves into the bulk flow, etc. In vehicular applications, emphasis on compactness and low weight discourages not only surface addition but also flow increase as the latter will result in an increase in the operating pressure, thus requiring the reinforcement of tube or duct walls. Rotation is one way to achieve heat transfer enhancement without suffering power losses in the ducts or weight and volume increases [4]. Moreover, heat transfer analysis for semi-closed rotating disk with source flow has been found to be available to cool high speed memory disks of electronic computers or impellers of cryogenic pumps [5].

In contrast, impinging jets have many applications, including the heating or drying of food, paper, chemicals, and the cooling of combustor components. They are also used in glass, metal, and plastic processing [6].

## **PROBLEM STATEMENT**

In recent years, promotion of heat transfer using a rotating disk system, with and without, impingement jet is a matter of great practical importance. The system cooling or heating by impinging jets has become an established technique, since relatively high local coefficients are obtained compared with those for non-impinging flows. This provides designer with a means for more effective control of the temperature of the system being heated or cooled.

## **LITERATURE SURVEY**

In (1987) Sparrow et al [7] described experimentally the heat transfer for a circular jet impingement on a confined disk. The experiments have been performed for a single circular jet impinging perpendicular to confined disk, with the spent air being collected in an annulus which surrounds the jet delivery tube. This configuration provides precise control of the surface area affected by the impinging jet and also assures complete collection of the spent air. During the course of the experiments, parametric variations were made of the dimensionless separation distance between the jet origin and the impingement disk, the ratio of disk diameter to the jet diameter, and the Reynolds number. They showed that the heat transfer coefficient at the impingement surface increased substantially with a decrease in the jet diameter. Furthermore, for smaller diameter jet, there was an optimum separation distance at which a maximum value of the heat transfer coefficient was achieved. Also, for a jet of larger diameter, the heat transfer coefficient decreased monotonically as the separation distance increased.

In (1988) Goldstein and Franchett [8] investigated experimentally the heat transfer to a jet impinging at different oblique angles to a plane surface. The angle between the jet axis and the surface was varied from 90 to 30 degrees. The experiment was performed for the distance of jet orifice to the plate ranging from four to ten orifice diameters. The results of measurements with the range of jet Reynolds number from 10000 and 30000. Both the jet orifice-to-plate spacing and the

jet angle influence the overall shape of the local Nusselt number profiles. The results were indicated that the location of the peak heat transfer is displaced somewhat from the intersection of the geometric axis of the jet with the plate surface, the displacement being a function primarily of impingement angle. For the normal jet the local distribution of the Nusselt number at spacing of  $L/D=4$  and  $L/D=6$  are very similar. The local Nusselt numbers near the peak for a spacing of  $L/D=10$  were about 20 percent less than the values measured for the spacing of 4 and 6. As the impinging jet was moved from normal, the local Nusselt number in the near peak region change. The rate at which these local Nusselt numbers vary with jet angle is dependent on the orifice-to-plate spacing. For the smaller spacing, the near-peak Nusselt numbers are not significantly effected by the initial decreases in the jet angle. For a given  $L/D$ , the slope of the local Nusselt numbers profile on the upstream increases with decreasing jet angles. The slope of local Nusselt number profile on the downstream decreases with decreasing jet angle. The profiles with smaller orifice-to-plate spacing and smaller jet angle have steeper profiles than the profiles with larger spacing and larger angles. Empirical equation was used to correlate the distribution of the local Nusselt number.

where:  $L$ : distance from jet orifice exit to heat transfer surface  
 $D$ : jet diameter

In (1989) Baughn and Shimizu [9] studied the heat transfer from a surface with uniform heat flux and an impinging jet. The study is concerned with the case of a single circular turbulent air jet at the ambient air temperature impinging on a flat stationary surface. The distribution of the Nusselt numbers along the surface is given for Reynolds number equal to 23750. It was shown that one of the most interesting distributions occurs when the jet is quite close to the surface ( $Z/D=2$ ). In this case, the maximum heat transfer is at the stagnation point; the heat transfer then has a minimum at  $r/D$  of approximately 1.3, and another maximum at approximately 1.8. As found by other investigators, the maximum stagnation point heat transfer occurs at a  $Z/D$  of approximately 6.

where:  
 $Z$ : distance for jet orifice exit to the surface.  
 $D$ : jet diameter

In (1989) Wang et al [10] solved analytically the conjugate problem associated with heat transfer between a laminar free impinging liquid jet and a laterally insulated disk with arbitrary

temperature or heat flux distribution. The local Nusselt number was found to depend upon the Prandtl number of the fluid, the ratio of the fluid conductivity to the solid conductivity, the aspect ratio of the thickness to the radius of the disk, and the prescribed temperature or heat flux distribution. When the solid conductivity is sufficiently high, the prescribed heat flux has little influence on the Nusselt number distribution because of high radial conduction. It was found that for a very thick disk, the effect of prescribed temperature or heat flux profile on the local heat transfer coefficient was negligible. For a thin disk, on the other hand, the prescribed temperature or heat flux profile has a considerable effect on the local heat transfer coefficient. Increasing the prescribed temperature or heat flux with  $r$  enhances the local Nusselt number while decreasing the prescribed temperature or heat flux with  $r$  reduces the local Nusselt number. The results also indicated that the local heat transfer coefficient becomes higher when the ratio of the fluid conductivity to the solid conductivity was larger.

In (1996) Cardone et al [11] measured the heat transfer on a rotating disk for a wide range of rotational Reynolds number values in the laminar, transition and turbulent flow regimes. Measurements were performed by making use of the heated thin-foil technique and by gauging temperature maps with an infrared scanning radiometer. The use of the radiometer is advantageous on account of its relatively good spatial resolution and thermal sensitivity and because it allows to easily make measurements down to very low local rotational Reynolds numbers. Data were obtained on two disks, having an external diameter of 300 mm and 450 mm. Heat transfer results presented in terms of Nusselt and Reynolds numbers based on the local radius, show that almost sudden rise of  $Nu$  where Reynolds number around  $2.5 \times 10^5$  is to be attributed to the onset of transition from laminar to turbulent flow. In the laminar zone, all the points fall around a straight line in the log-log plan down to very low Reynolds numbers. In the fully turbulent regime data fit the relation:

$$Nu = 0.0163 Re_r^{0.8}$$

In (1997) Shevchuk and Khalatov [12] improved an approximation of the radial velocity profile observed in a turbulent boundary layer on a rotating disk, that proposed on the basis of the assumption that the tangent of the swirl angle of the flow is described by a quadratic function. The quadratic approximation used in the calculations for the tangent of the

flow swirl angle in the boundary layer provides a better agreement between the calculated and experimental profiles of the radial velocity. The authors succeeded to obtain the solutions to the equations describing the boundary layer and it was compared the solutions with those obtained by other authors for different cases such as, a radial air flow around the disk and a rotation of the fluid by the law of typical of a solid body.

In (1999) Shevchuk and Khalatov [13] Applied an approximation of the distribution of the radial velocity in turbulent flow in rotating disk systems for calculating the thermal boundary layer, based on a quadratic equation for the tangent of the flow swirl angle. The temperature distribution was approximated by a power law equation. Analytical results were obtained for the temperature distributions and the ratio of the thickness of the thermal and velocity boundary layers. The authors obtained the solution of the integral equations of the thermal boundary layer under superposed radial flow conditions and with solid body rotation of the liquid, also it was compared this solution with published analytical data.

In (1999) Shevchuk [14] presented results of simulation of turbulent heat transfer and hydrodynamics over a free rotating disk using an integral method based on power law velocity and temperature profiles and three different laws for the tangent of the flow swirl angle. It appeared that a quadratic correlation of the tangent of the flow swirl angle was the most proper one for a free rotating disk. Resulting equation for the Nusselt number was in a better agreement with experimental data of different authors.

In (1999) Shevchuk [15] discussed a turbulent centrifugal flow in a gap between parallel rotating disks, in a special case that the flow tangential velocity at the inlet is less than or equal to the tangential velocity of the disks. The equations of a boundary layer were numerically solved by an integral method based on a power approximation of the tangential velocity and a quadratic approximation of the tangent of the flow swirl angle. The developed method allowed achieving significantly better agreement between the results of calculation and the available experimental data than did various methods.

In (2001) Shevchuk [16] developed integral method for predications of heat transfer near a free rotating disk for the Prandtl numbers less or equal to unity including the case of

Prandtl number tending to zero. The proposed integral method allowed obtaining an approximate analytical solution for the Nusselt number for both laminar and turbulent flow cases, and this solution is significantly more accurate than other known relationships, especially at almost zero or negative gradients of the wall temperature. It was shown that the accuracy of the proposed approximate analytical solution for the Nusselt number is substantially higher and the Nusselt number computed from the proposed formula does not exceed 3.1% with respect to the exact numerical solution.

In (2001) Usha and Ravindran [17] examined numerically the development of flow and heat transfer characteristics of a heat conducting fluid film on a rotating disk for a wide range of Reynolds numbers. They succeeded to solve the Navier-Stokes equations and the corresponding energy equation numerically by a finite difference method. The results were analyzed for different values of the cooling or heat dissipating parameter and the Prandtl number. The formulation was based on the following assumptions:

- i) The free surface is initially planar and remains planar with spinning.
- ii) The radius of the disk is much larger than the film thickness, so the edge effect can be neglected.
- iii) Thermal stress on the free surface is neglected.
- iv) All physical properties like viscosity, surface tension, etc, are constant and independent of temperature.

They observed that inertial forces have a significant influence on the flow characteristics, film thickness, and on the axisymmetric cooling of the rotating disk. It was further noted that the rate of thinning of the fluid film is strongly influenced by the cooling parameter and Prandtl number, when the cooling parameter increase or Prandtl number decreases, the film thickness increase. And the film thickness decreases with time for fixed angular velocity of the rotating disk. The solution was useful in validating more realistic analytical or numerical models of film cooling on a spinning disk.

## **PROJECT DESCRIPTION**

This work is aimed to study the behavior of flow and heat transfer characteristics on the surface of a rotating disk, due to the circular jet of liquid impinging at the center of the disk.

A theoretical model is introduced based on heat balance and energy equations developed for a rotating disk heated by a

constant heat flux and the free surface are subjected to surrounding.

Local Nusselt number and non-dimensional bulk liquid temperature are found for parameters such as Prandtl number and Reynolds number.

The investigation are limited to impinging steady, incompressible, and laminar flow of a liquid in a thin film on the surface of a rotating disk. Also, the liquid in this film will be moved radially outward along the disk under the action of the centrifugal force.

### HEAT TRANSFER MODEL DUE TO IMPINGING LIQUID JET

A model is developed to describe the flow and heat transfer characteristics of liquid film flowing over the surface of a rotating disk, due to the circular jet of liquid impinging at the center of the disk. The present study is concerned with the case of a single circular liquid jet at the liquid temperature ( $T_o$ ), the liquid film having a mass flow rate, and bulk temperature ( $T_b$ ).

The rotating disk is heated by a constant heat flux, and the free surface is subjected to the surrounding and the heat transfer is due convection according to Newton's cooling law's.

#### 1. HEAT BALANCE EQUATION

In order to formulate a mathematical model which enables to estimate the temperature distribution and to find the effect of various parameters on heat transfer process, it is found that the heat balance equation must be used and as shown in Figure 1, it can be written as:

$$\dot{m}c_p(T_b + dT_b) - \dot{m}c_p T_b = [q'' - h(T_b - T_\infty)]2\pi r dr \quad \dots(1)$$

This equation can be rearranged in the following form, to make the integration possible:

$$\frac{\dot{m}c_p dT_b}{[q'' - h(T_b - T_\infty)]} = 2\pi r dr \quad \dots(2)$$

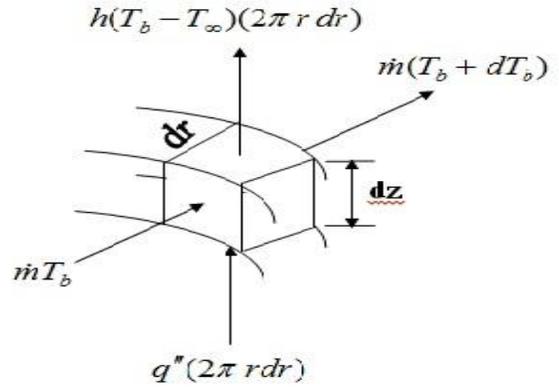
In order to find the bulk temperature distribution in the direction parallel to the disk surface, Equation (2) integrated subjected to boundary conditions:

$$T_b = T_o \quad \text{at} \quad r = r_o, \quad T_\infty = T_o \quad \dots(3)$$

result in the following equation:

$$T_b = -\frac{q''}{h} \left[ \text{EXP} \left[ -\frac{\pi r_o^2 h}{\dot{m}c_p} \left( \frac{r^2}{r_o^2} - 1 \right) \right] - 1 \right] + T_o \quad \dots(4)$$

Thus it is possible to obtain the bulk temperature of the liquid at any position r.



**Figure 1:** Overall heat balance applied to control volume of a thin film flow over the surface of a rotating disk due to impinging liquid.

#### 2. ENERGY EQUATION

Energy equation is obtained from the first law of thermodynamics, which is an expression of the conservation of energy, it states that the amount of heat added to a fluid is equal to the change in it's energy plus any work done by the fluid [18].

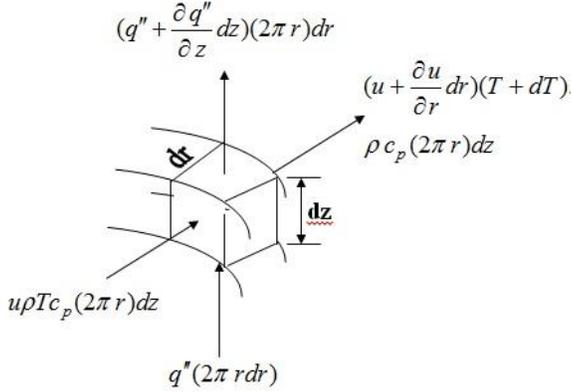
For an incompressible fluid the energy balance is determined by the internal energy, the conduction of heat, the convection of heat with the stream, and the generation of heat through friction. In a compressible fluid there is an additional term due to the work of expansion (or compression) when the volume is changed [19].

The forgoing analysis considered the fluid dynamics of a laminar boundary layer flow system. Consider the element control volume shown in Figure 2. The following assumption are used to simplify the analysis:

- Incompressible steady flow

- Constant viscosity, thermal conductivity, and specific heat of the fluid
- Negligible heat conduction in the direction of flow
- Negligible heat generation due to the friction

....(14)



**Figure 2:** Elemental control volume for energy analysis of laminar boundary layer.

The energy balance becomes:

$$\rho c_p \frac{\partial}{\partial r} [uT] = \frac{\partial q''}{\partial z} \quad \text{....(5)}$$

By using Fourier law of heat conduction equation (5) becomes:

$$\frac{\partial}{\partial r} [uT] = \alpha \frac{\partial^2 T}{\partial z^2} \quad \text{....(6)}$$

In order to simplify the solution of the above equation without losing the accuracy it is possible to assume that the gradient of (uT) in the direction of flow is equal to the gradient of (uT)<sub>b</sub> in the same direction. This special case is possible especially for thin liquid film.

By using the above simplification it is possible to write equation (6) as:

$$\frac{d}{dr} [uT]_b = \alpha \frac{\partial^2 T}{\partial z^2} \quad \text{....(7)}$$

with boundary conditions:

$$\frac{\partial T}{\partial z} = -\frac{q''}{k} \quad \text{at } z = 0 \quad \text{....(8)}$$

In order to obtain the temperature profile in the liquid layer on the surface of a rotating disk, equation (7) can be integrated subjected to the value of temperature  $T = T_w$  at  $z=0$ , and after performing some arrangement the result can be formulated as:

$$T = \frac{1}{\alpha} \frac{d}{dr} [uT]_b \frac{z^2}{2} - \frac{q''}{k} z + T_w \quad \text{....(9)}$$

### 3. TEMPERATURE DISTRIBUTION

The definition of the bulk temperature which is given by:

$$T_b = \frac{\int_0^\delta uT dz}{\int_0^\delta u dz} \quad \text{....(10)}$$

The velocity profile needed in the above equation to proceed with temperature distribution is assumed in the following form [20],

$$u = \frac{\omega^2 r \delta^2}{2\nu} [2(z/\delta) - (z/\delta)^2] \quad \text{....(11)}$$

The equation that represented the temperature difference between the bulk and the disk surface temperature yields:

$$T_b - T_w = \frac{9}{40} \frac{1}{\alpha} \frac{d}{dr} [uT]_b \delta^2 - \frac{5}{8} \frac{q''}{k} \delta \quad \text{....(12)}$$

The volumetric flow rate is defined as:

$$Q = \bar{u} * A_c \quad \text{....(13)}$$

where  $\bar{u}$  is the average velocity and  $A_c$  is the cross section area and are equal to,

$$\bar{u} = \frac{1}{\delta} \int_0^\delta u dz \quad \text{....(14)}$$

$$A_c = 2\pi r \delta \quad \text{....(15)}$$

the average velocity can be written as:

$$\bar{u} = \frac{\omega^2 r \delta^2}{3\nu} \quad \dots(16)$$

Substitution of Equation (16) into Equation (13) to indicate the relation between the flow rate and the film thickness, the result can be written in the following form:

$$\delta = \delta_i \left(\frac{r}{r_o}\right)^{-2/3} \quad \dots(17)$$

Differentiating Equation (17) and (4) to obtain the film thickness and bulk temperature profiles with respect to radial coordinate the result will be:

$$T_b - T_w = \left(\frac{r}{r_o}\right)^{-2/3} \left[ \frac{9}{40} \frac{q'' \delta_i}{k} \text{EXP}\left[-\frac{\pi r_o^2 h}{\dot{m} c_p} \left(\frac{r^2}{r_o^2} - 1\right)\right] - \frac{5 q'' \delta_i}{8 k} \right] + \frac{3 \dot{m} c_p \delta_i q''}{80 \pi k r_o^2 h} \left(\frac{r}{r_o}\right)^{-8/3} \left[ \text{EXP}\left[-\frac{\pi r_o^2 h}{\dot{m} c_p} \left(\frac{r^2}{r_o^2} - 1\right)\right] - 1 \right] - \frac{3 \dot{m} c_p \delta_i T_o}{80 \pi k r_o^2} \left(\frac{r}{r_o}\right)^{-8/3} \quad \dots(18)$$

The above equation gives the temperature difference between the bulk and wall temperature at any local value of r.

Using the dimensionless variables:

$$\left. \begin{aligned} T_b^+ &= \frac{(T_b - T_o)h}{q''} \\ T_w^+ &= \frac{(T_w - T_o)h}{q''} \\ Nu &= \frac{h\delta}{k} \\ Pr &= \frac{\mu c_p}{k} \\ Bi &= \frac{h r_o}{k} \\ Re &= \frac{4\dot{m}}{2\pi r_o \mu} \\ R &= \frac{r}{r_o} \end{aligned} \right\} \quad \dots(19)$$

The temperature distribution in dimensionless form can be rewritten as,

$$T_w^+ - T_b^+ = -\frac{9}{40} Nu \text{EXP}\left[-\frac{2Bi}{Re Pr} (R^2 - 1)\right] + \frac{5}{8} Nu - \frac{3 Re Pr Nu}{160 Bi R^2} \left[ \text{EXP}\left[-\frac{2Bi}{Re Pr} (R^2 - 1)\right] - 1 \right] + \frac{3 Re Pr Nu}{160 Bi R^2} \left(\frac{T_o h}{q''}\right) \quad \dots(20)$$

## RESULTS AND DISCUSSIONS

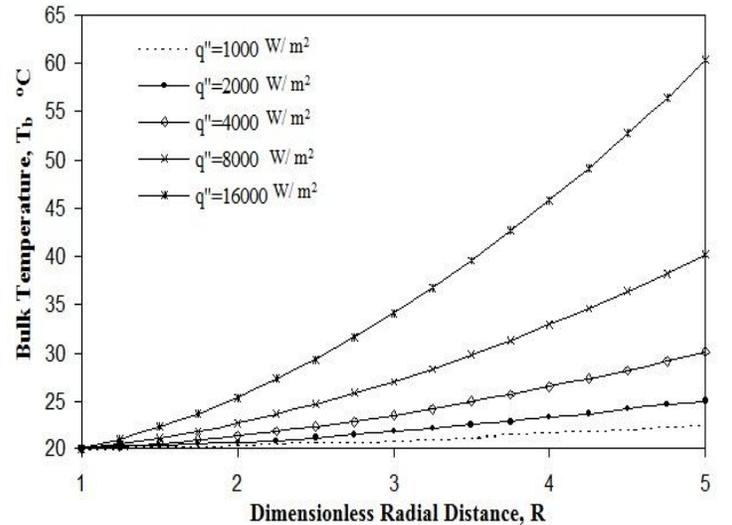
Attention is focused on determination of parameters, which are considered necessary for describing the behavior of the developed model and explaining the heat transfer characteristics. These parameters are composed of the bulk temperature of the liquid film, and the local Nusselt number along the flow. The calculation of each of the above mentioned parameters is taking into consideration the effect of several important parameters such as heat flux, volumetric flow rate, and heat transfer coefficient of surrounding. The results of the calculation above are described below.

### 1. EFFECT OF HEAT FLUX

Effect of heat flux on the heat transfer characteristics is shown in figures 3–5. Each figure composed of five plotted curves for the selected values of heat flux. The effect of heat flux can be explained for each of the following result as:

#### i- Effect on bulk temperature

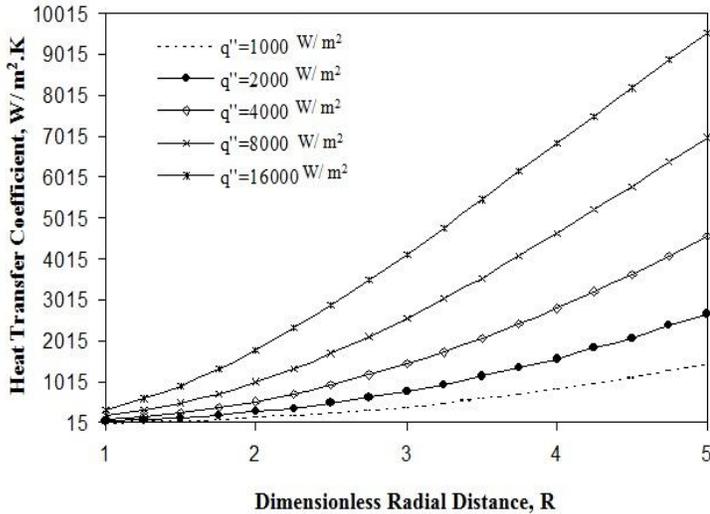
Figure 3 reflect the effect of heat flux on the bulk temperature along the flow. In this figure, for a fixed value of heat flux, the increment in radial distance means exposing the liquid film to higher quantity of heat due to increasing the exposed area and the fixed heat flux. This behaviour caused an increase in the bulk temperature of the liquid film. On the other hand, increasing heat flux cases the bulk temperature at fixed position to increase in the same manner.



**Figure 3:** Effect of heat flux on bulk temperature distribution along a rotating disk with,  $Q=1$  (1/min),  $h=50$  (W/ m<sup>2</sup>.K) and  $r_o=0.05$  m

**ii- Effect on heat transfer coefficient**

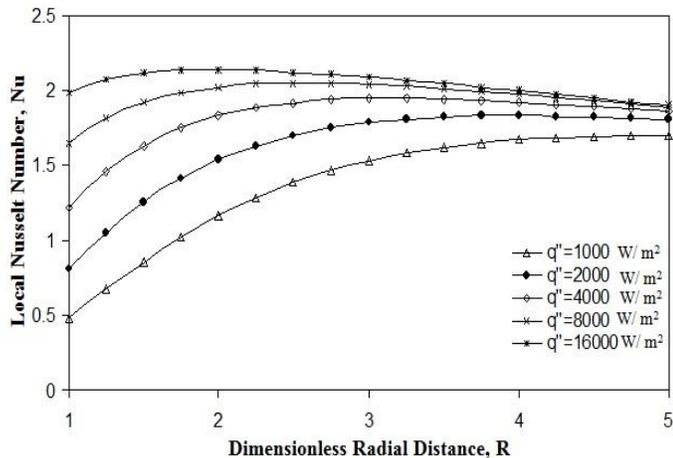
The behavior of heat transfer coefficient with heat flux is shown in figure 4. From this figure it is clear that, for a fixed heat flux the heat transfer coefficient increase along the flow. This is due to decrease in the film thickness with increasing the radial distance.



**Figure 4:** Effect of heat flux on heat transfer coefficient of the liquid layer along a rotating disk with  $Q=15$  (1/min),  $h=50$ (W/m<sup>2</sup>.K) and  $r_o=0.05$  m

In contrast, it is interesting to note that the value of heat transfer coefficient increase with increasing heat flux. Consequently, local Nusselt number increase with increasing the heat flux as illustrates in figure 5. This behaviour can be explain from the definition of heat transfer coefficient.

$$h = \frac{q''}{T_w - T_b} \quad \dots(21)$$



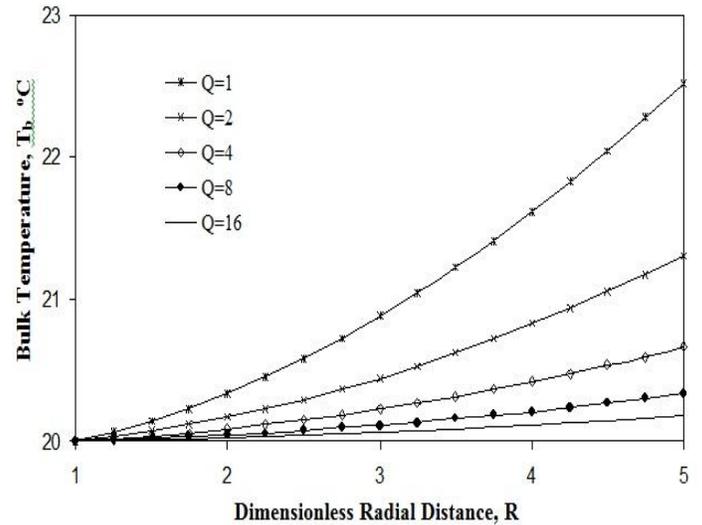
**Figure 5:** Effect of heat flux on local Nusselt number along a rotating disk with  $Q=1$  (1/min),  $h=50$  (W/m<sup>2</sup>.K) and  $r_o=0.1$

**2. EFFECT OF FLOW RATE**

Figures 6-8, clarify the effect of flow rate on heat transfer characteristics. In these figures the results are plotted for various values of flow rate, while the other parameters are kept constant. The effect of flow rate can be explained as:

**i- Effect on bulk temperature**

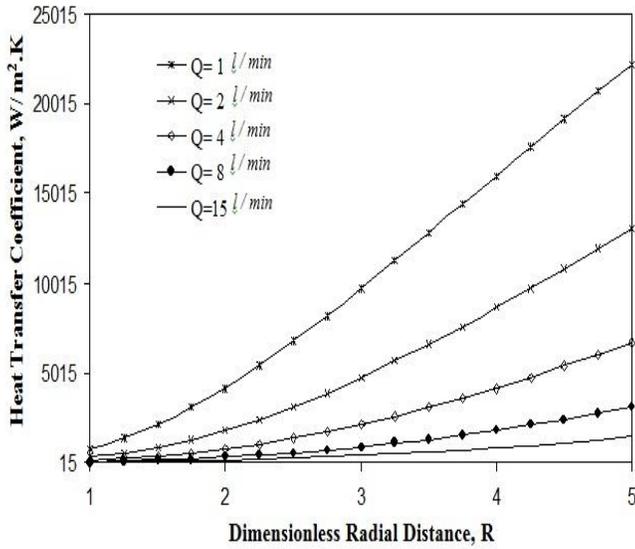
The effect of flow rate on bulk temperature is presented in Figure 6. In this figure the increments of flow rate lead to decrease the bulk temperature. This behaviour can be explain as, for a constant position along the flow the value of heat transfer rate is constant since the exposed area is constant, so from heat balance equation the increment in flow rate is associated with lower bulk temperature for constant properties of the liquid layer.



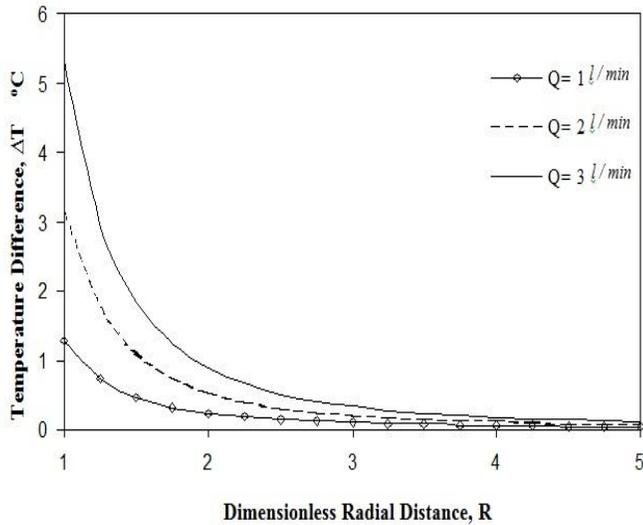
**Figure 6:** Effect of flow rate on bulk temperature distribution along a rotating disk with,  $q=1000$ (W/m<sup>2</sup>),  $h=50$ (W/ m<sup>2</sup>.K) and  $r_o=0.05$ m

**ii- Effect on heat transfer coefficient**

Figure 7 explain the relation between the flow rate and heat transfer coefficient of the liquid film. It is evident that, for a fixed value of flow rate the heat transfer coefficient increases with increasing radial distance. This is due to decrease of the liquid film thickness with increasing the radial distance casing an increase in the temperature different ( $T_w - T_b$ ) as shown in figure 8.



**Figure 7:** Effect of flow rate on heat transfer coefficient of the liquid layer along a rotating disk with  $q''=1000(W/m^2)$ ,  $h=50(W/m^2.K)$  and  $r_o=0.05$  m

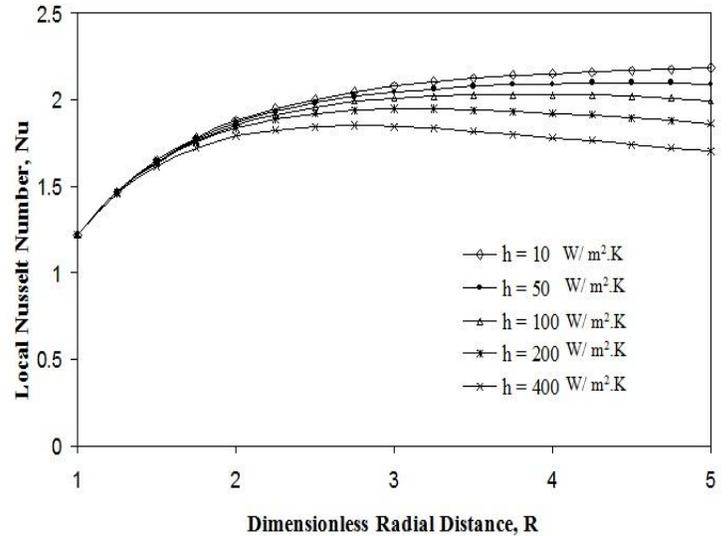


**Figure 8:** Effect of flow rate on temperature difference ( $T_w - T_b$ ) °C along a rotating disk with  $q''=1000(W/m^2)$ ,  $h=50(W/m^2.K)$  and  $r_o=0.05$  m

### 3. EFFECT OF HEAT TRANSFER COEFFICIENT OF SURROUNDING

In figure 9, the local Nusselt number is plotted against the radial distance for different values of heat transfer coefficient of surrounding. From this figure, it is obvious that the local Nusselt number decreases with increasing heat transfer coefficient of surrounding. This behavior can be explained as,

since the increment in heat transfer coefficient of surrounding lead to increase the temperature difference between the surface and the bulk temperature and that lead to decrease the heat transfer coefficient of the liquid layer.

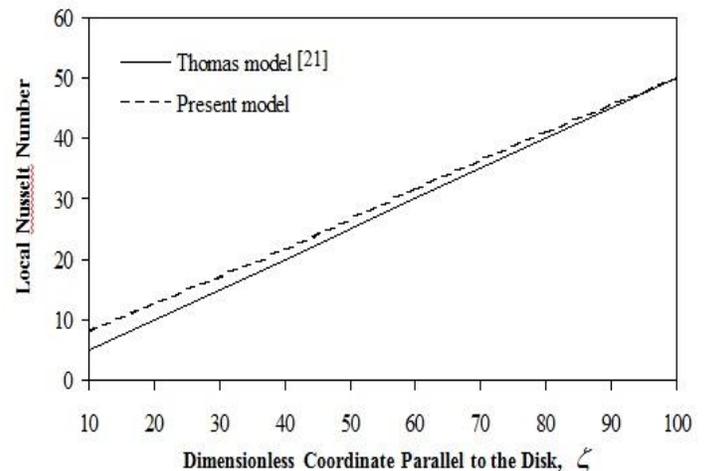


**Figure 9:** Effect of heat transfer coefficient surrounding on local Nusselt number along a rotating disk with  $q''=16000(W/m^2)$ ,  $Q=1(l/min)$  and  $r_o=0.05$

### COMPARISON WITH OTHER MODEL

The comparison is used to show how the results obtained by the present analysis for the developed model of laminar flow along a rotating disk is far related to the other models obtained by other authors.

The result of local Nusselt number in the present model are compared with results in ref.[21] in Figure 10. From this figure the results obtained in the present work show good agreement with these works. The parameter used in this comparison is the local Nusselt number at different position along the disk.



**Figure 10:** Comparison of the present model with Thomas model for  $Ro=10-2$  and  $\zeta_i = 0.2$

## CONCLUSIONS

A theoretical model is developed for laminar flow on the surface of a rotating disk, due to the individual impingement jet in which the jet flow is constrained to exit in a single direction and impinging the surface of the disk at the center.

In the present model of heat transfer due to impingement liquid jet, it is found that the bulk temperature ( $T_b$ ) and Nusselt number (Nu) are influenced by several important parameters like, Prandtl, Reynolds, and Biot numbers. The local Nusselt number obtained from the present model was compared with the available previous analytical data to assess the validity of the present theoretical models and the agreement between the result was acceptable.

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